# EVALUATION OF DIFFERENT OPTIMIZATION MODELS FOR SCHEDULING SILVICULTURAL OPERATIONS

André Laroze, Paulina Pinto, Fernando Muñoz[1]

ABSTRACT.—Currently in Chile, silvicultural tasks present the highest level of labor instability among forestry activities. Due to their seasonal character and short duration, a continuous change of nonspecialized workers is generated. To help reverse this situation it is necessary to consider cost and labor stability simultaneously to determine in advance the number of workers required to perform such tasks. The objective of this study was to develop and compare several models for solving this problem, based on different objective functions and constraint sets. The results obtained indicate that the analyzed models can contribute to effective scheduling of a company's silvicultural activities at an operational level.

## INTRODUCTION

The forestry sector is currently one of the most dynamic agents of the Chilean economy. In an open economy, however, the productive factors constantly change and the activities become more competitive and less profitable. This situation has led forestry companies to develop new techniques for improving the return on their investments by means of increasing yields, optimizing production systems, and minimizing operational costs.

Mathematical programming models can find good solutions for a variety of problems such as selection of management regimes, log merchandising, and transportation scheduling. However, the problems analyzed by forestry companies have mainly dealt with efficient resource allocations at a strategic (Barros and Weintraub, 1982; García 1984) and tactical level (Laroze and Greber 1991; Weintraub et al. 1994). Simulation and optimization techniques have been used less frequently for solving operational problems; their absence in scheduling silvicultural activities is particularly notorious (Muñoz and Andalaft 1991).

Silvicultural interventions present the highest level of labor instability within the Chilean forestry sector. The seasonal character and short duration of these tasks translates into a continuous rotation of nonspecialized workers. This situation could be reversed by considering aspects of cost minimization and labor stability simultaneously when scheduling silvicultural activities; that is, keeping the work force as regular as possible and thus reducing the normal monthly fluctuations without affecting operational costs. Such a planning effort can improve the use of forest camps, increase the companies' administrative efficiency, and achieve a labor stability that should stimulate a worker-training program by the forestry contractors.

This study developed several optimization models to schedule a company's silvicultural tasks considering a 1-year planning horizon. The lack of proven models for solving this type of problem made it necessary to design and evaluate alternate formulations. The approach of implementing different objective functions and constraint sets and later comparing the results in terms of cost and labor stability allowed us to establish a process for generating cost-effective solutions at an operational level.

[1] André Laroze, Assistant Professor, Department of Forest Sciences, P. Universidad Católica de Chile. Paulina Pinto, Research Assistant, Proyecto FONDECYT # 1960248. Fernando Muñoz, Manager, Department of Forest Management, Gerencia Concepción, Forestal Mininco S.A.

## BASIC INFORMATION

Forestal Mininco's Department of Forest Management—Concepcion Region provided the data corresponding to the silvicultural activities. Information on 20 tree farms that represented diverse conditions was extracted from the 1995 annual plan. For each tree farm, the data consisted of the area, cost, labor productivity, and feasibility periods for eight silvicultural interventions. A 12-month planning horizon was considered. The extension of the planning horizon, number of tree farms, and number of tasks selected correspond to a real-size problem representative of the decisions carried out at a district level (the company's basic administrative unit).

## PROBLEM MODELING

Based on the annual program established for the silvicultural activities, it is possible to use mathematical programming techniques for efficiently determining the work force required for performing each task at each tree farm on a monthly basis. The input required consists of the periods in which it is feasible to perform the tasks, the expected labor productivity and cost, and the available budget.

Since the problem of scheduling silvicultural tasks was not clearly defined, we evaluated several models generated as a combination of different objective functions and constraint sets. Thus, one could analyze multiple scenarios and compare the optimal solutions obtained by the different formulations. The following section presents the different objective functions and restrictions considered.

### Objective Functions

- *Minimization of total cost* finds the solution that minimizes the total cost incurred for carrying out all of the silvicultural interventions in every tree farm.
- *Minimization of total labor* generates the solution that minimizes the work force required to execute all of the silvicultural tasks.
- *Minimization of the work force variance* reduces the variance in the number of person days hired throughout the season, and therefore, is a proxy for maximizing labor stability.
- *Minimization of the maximum work force required per* lowers the highest labor requirements for any particular month. Therefore, it indirectly attempts to reduce the total number of workers required during the season, and to standardize the work load in the different periods. Restrictions that ensure a positive difference between the objective value and the number of person days hired in every period of the season complement this function.

### Constraints

- *Annual program (A)* forces the solution to perform the annual program of silvicultural activities in every tree farm. It is implemented in all models.
- *Range of allowable person days per period (R)* requires that the number of person days per month that neither exceeds a maximum boundary nor is lower than a minimum target, taking into account all the tasks in the different tree farms. This range regulates the work load distribution in absolute terms.
- *Work force fluctuation (F)* keeps the relative differences in the number of person days hired in consecutive months within a predetermined percentage.
- *Minimum and maximum monthly size per activity at each tree farm (S)* force each silvicultural task to have a minimum extension, defined by a practical limit that makes its execution convenient, and not to exceed a maximum size, set by an operational monitoring limit.

- *Continuity condition (C)* encourages the tasks to be carried out without interruptions by associating a cost to each beginning and ending of a task. If the assigned costs are high enough, the continuity conditions become implicit restrictions.
- *Maximum total cost (P)* keeps the solution from exceeding the available budget for executing the total number of tasks in the different tree farms.

## ANALYSIS OF RESULTS

A large number of models can be defined by combining different objective functions with diverse groups of restrictions. For this study, however, the analysis was focused on the issues that are described below.

### Basic Parameters Used for Restrictions

Table 1 lists characteristics of two optimization problems considered in this study. Table 2 presents the base values of the different restrictions that were considered for the problems. A maximum variation of ±10 percent is accepted in the fluctuation of the number of person days hired in consecutive months. As for the work force allowed to be hired each month, the minimum was set at 5,850 and the maximum at 7,150 person days for all the periods. These values correspond to approximately 10 percent variation regarding the average number of hired person days per month (6,517), according to the solution of the variance minimization objective function subject to the execution of the operative plan. For each task, the minimum and maximum size of a monthly silvicultural intervention carried out in a tree farm are indicated in this table, as well as the constant for determining the condition of continuity (0.1 hectares) and the maximum total cost. This last value, which is used as a reference, corresponds to the solution of the model for minimizing total cost subject to the execution of the annual plan.

**Table 1.—Characterization of two optimization problems**

| Criteria | Problem A | Problem B |
|---|---|---|
| Objective Function | Minimize total cost | Minimize total cost |
| Type of restrictions | Annual program (A) | Annual program (A)<br>Range of allowable person days per period (R)<br>Work force fluctuation (F)<br>Minimum and maximum monthly size per activity at each tree farm (S)<br>Continuity condition (C) |
| Number of variables | 364 | 1456 |
| Number of complete variables | 0 | 1092 |
| Number of restrictions | 100 | 1354 |
| Number of nonlinear restrictions | 0 | 0 |
| Coefficients other than zero | 3213 | 10195 |

**Table 2.—Base parameters used for restrictions**

| Silvicultural Activity | Area | |
|---|---|---|
| | Minimum | Maximum |
| | - - - - *hectares* - - - - | |
| A | 15 | 85 |
| B | 15 | 100 |
| C | 20 | 80 |
| D | 20 | 100 |
| E | 10 | 150 |
| F | 25 | 225 |
| G | 25 | 225 |
| H | 20 | 250 |

Continuity constant [ha]: 0.1
Maximum total budget: $1,675,525

## Comparison of Objective Functions

This analysis was carried out to compare the behavior of the four basic models. These models consist of each one of the objective functions and only consider the restriction for executing the silvicultural interventions according to the company's operative targets. Furthermore, such solutions are compared with those that result from including an additional constraint that regulates the monthly minimum and maximum area of the tasks. Thus, it is possible to evaluate the effect of these restrictions—in terms of cost and labor stability—according to the type of objective function.

With regards to the analysis of the basic models, Table 3 shows that the objective function of total cost minimization, which is the most efficient in economic terms, presents a great monthly variation in the budget and work force required. The objective function for minimization of total person days also generates a high operative instability, and even though it obtains a better average productivity, it induces an increase in the total cost. This situation is due to the fact that periods of better physical yield in the tasks do not necessarily coincide with lower operational costs. Upon comparing both solutions, an increase of 3.6 percent with regards to the total cost (from thousand-$ 1,676 to 1,736) and a decrease of 3.5 percent in the number of required person days (from 76,490 to 73,793), is observed.

Figure 1 presents the work force distribution per month corresponding to the solution obtained for each of the basic models. One can observe that the objective functions minimizing the maximum monthly work force (MMW) and the variance in the number of person days generate a homogeneous distribution of work all year round. However, this greater labor stability implies an important increase in costs, which affects variance minimization more. In particular, the latter objective function, which is insensitive to cost and total number of person days, achieves greater labor stability by means of an inefficient assignment of tasks. That is, in order to even out the work force in the months of less activity, it allocates the tasks of poorest productivity. Consequently, this solution implies an increase of 8.7 percent in costs and 5.4 percent in person days, compared to the solution for the objective functions of minimizing total cost and person days, respectively.

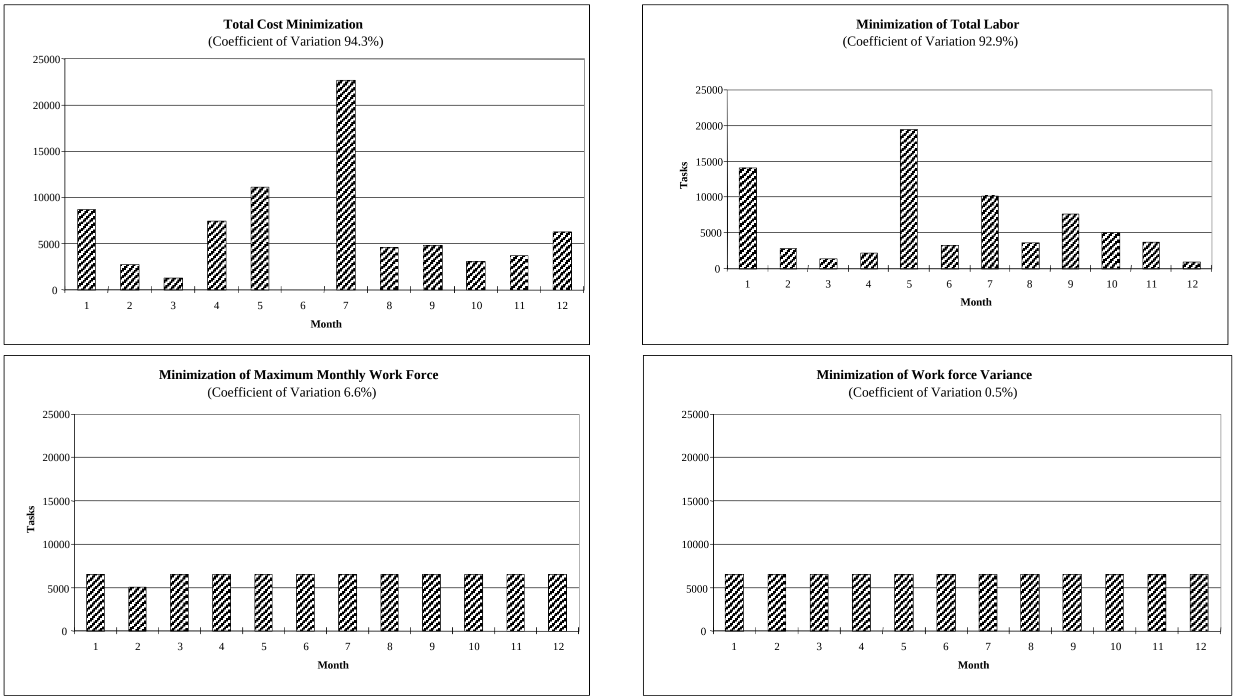


**Figure 1.—Monthly work load distribution according to selected objective function.**

Upon analyzing the solutions at a tree farm level, it was observed that the objective functions of minimizing costs and person days concentrate the completion of each task only in the month that is most convenient. In several cases, these solutions exceed the contractors' actual work load capacity. On the other hand, the objective functions of minimizing MMW and variance generate solutions that imply levels of very low activity per month on some tree farms that do not justify keeping a task open. To solve this inconvenience, a constraint was added that limits the minimum and maximum area that a task should have on a property in a month. Table 3 shows that this additional restriction generates an important increase of the operational costs for the objective functions of minimizing costs and person days, but it also reduces the variability of monthly activity levels. In the case of the other two objective functions, the effect of the added constraint had a lower impact on costs but decreased labor stability.

**Table 3.—Comparison of objective functions**

| Analyses | | Total | Average Monthly Results | | | | | | Solution Proces | |
|---|---|---|---|---|---|---|---|---|---|---|
| Minimize: | Subject to: | | Mean | Median | Minimum | Maximum | Range | CV | Iterations | Tim |
| Cost | | | - - - - - - - - - - - - - - - - *thousand dollars* - - - - - - - - - - - - - - - - | | | | | *percent* | | |
| Total Cost | A | **1676** | 140 | 92 | 0 | 444 | 444 | 87.2 | 1 | 00:0 |
| Total Labor | A | 1736 | 145 | 83 | 17 | 469 | 453 | 91.6 | 1 | 00:0 |
| MMW | A | 1794 | 149 | 145 | 118 | 180 | 62 | 13.5 | 221 | 00:0 |
| Work force Variance | A | 1821 | 152 | 147 | 121 | 180 | 58 | 11.9 | 220 | 00:0 |
| Total Cost | A, S | **1709** | 142 | 109 | 91 | 311 | 220 | 48.7 | 954 | 00:0 |
| Total Labor | A, S | 1759 | 147 | 133 | 56 | 303 | 246 | 49.4 | 991 | 00:0 |
| MMW | A, S | 1816 | 151 | 149 | 99 | 183 | 84 | 16.3 | 26802 | 00:0 |
| Work force Variance | A, S | 1825 | 152 | 145 | 126 | 183 | 58 | 14.1 | 250000 | 24:1 |
| Work force | | | - - - - - - - - - - - - - - - - - - *person days* - - - - - - - - - - - - - - - - - - | | | | | | | |
| Total Cost | A | 76490 | 6374 | 4714 | 0 | 22688 | 22688 | 94.3 | | |
| Total Labor | A | **73793** | 6149 | 3606 | 880 | 19450 | 18570 | 92.9 | | |
| MMW | A | 77088 | 6424 | 6546 | 5077 | **6546** | 1469 | 6.6 | | |
| Work force Variance | A | 78208 | 6517 | 6517 | 6486 | 6551 | 65 | **0.5** | | |
| Total Cost | A, S | 78405 | 6534 | 5141 | 3323 | 16345 | 13023 | 58.4 | | |
| Total Labor | A, S | **75915** | 6326 | 6255 | 2703 | 12165 | 9463 | 48.3 | | |
| MMW | A, S | 78403 | 6534 | 7078 | 4263 | **7078** | 2815 | 15.1 | | |
| Work force Variance | A, S | 78519 | 6543 | 6534 | 5931 | 7147 | 1216 | **8.5** | | |
| Productivity | | | - - - - - - - - - - - - - - - *person days/hectare* - - - - - - - - - - - - - - - | | | | | | | |
| Total Cost | A | | 4.13 | 3.92 | 1.89 | 10.50 | 8.61 | 73.2 | | |
| Total Labor | A | | 3.99 | 3.51 | 2.10 | 11.00 | 8.90 | 80.7 | | |
| MMW | A | | 4.16 | 3.88 | 3.07 | 9.15 | 6.08 | 40.4 | | |
| Work force Variance | A | | 4.23 | 3.95 | 3.07 | 6.30 | 3.23 | 21.6 | | |
| Total Cost | A, S | | 4.24 | 4.23 | 2.84 | 9.45 | 6.61 | 50.1 | | |
| Total Labor | A, S | | 4.10 | 4.08 | 2.68 | 9.85 | 7.18 | 54.9 | | |
| MMW | A, S | | 4.24 | 4.15 | 3.28 | 8.20 | 4.92 | 33.0 | | |
| Work force Variance | A, S | | 4.24 | 4.17 | 3.28 | 6.75 | 3.48 | 23.0 | | |

A = Annual program; S = Minimum and maximum size per activity at each tree-farm; CV = variation coefficient.

The solution time is negligible for the basic models, including that of variance minimization, which is the only nonlinear programming model. Regarding the problems with integer variables, a significant increase in the number of iterations took place for MMW model, but the solution time did not exceed 10 minutes on a Pentium Intel-166 MHz computer. However, with the variance minimization problem, a nonlinear integer model, the processing time exceeded 24 hours even when the solution corresponding to the MMW model was used as an initial starting point. Therefore, the best feasible solution obtained within a limit of 250,000 iterations was selected in this case.

## Effect of the Restriction Levels

For the objective function of minimizing the total cost, models with three types of constraints were formulated: completion of tasks, range of person days, and maximum work force fluctuation. The analysis consisted of defining diverse restriction levels for the last two types of constraints, where each level corresponds to a certain value for the parameters associated with the constraint. These tests enabled us to assess the effect of the different requirements of work force distribution stated in the problem on the costs and labor productivity.

Table 4 shows that, by restricting the fluctuation in the number of person days between consecutive periods, one can achieve a considerably more even level of activity upon using the objective function of minimizing the total cost. However, the changes in the number of person days represent a trend: the difference accumulated in an interim of 6 months is close to 35 percent for a tolerated fluctuation of 10 percent (4,555, 7,336, and 5,374 person days for months 1, 6, and 12, respectively).

**Table 4.—Effect of the restriction levels on cost, work force, and productivity when attempting to minimize total cost**

| | Monthly Results | | | | | | | | | | | | General Results | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Restrictions: | | | | 4 | | ; | ' | 8 | 9 | 0 | 1 | .2 | al | an | CV |
| | - - - - - - - - - - - - - - - - - - - - - - - - - - -*thousand dollars* - - - - - - - - - - - - - - - - - - - - - - - - - - - | | | | | | | | | | | | | | *cent* |
| A | 208 | 79 | 39 | 182 | 270 | 0 | 444 | 81 | 104 | 80 | 70 | 119 | **1676** | 140 | 87.2 |
| A, $F_1$ | 99 | 112 | 147 | 159 | 169 | 200 | 201 | 141 | 134 | 139 | 96 | 109 | **1707** | 142 | 25.1 |
| A, $F_B$ | 109 | 117 | 153 | 154 | 160 | 178 | 161 | 163 | 154 | 150 | 116 | 103 | **1717** | 143 | 17.4 |
| A, $F_2$ | 120 | 136 | 162 | 155 | 157 | 164 | 152 | 144 | 158 | 137 | 135 | 117 | **1738** | 145 | 10.9 |
| A, $R_1$ | 174 | 113 | 44 | 177 | 170 | 173 | 158 | 132 | 157 | 150 | 151 | 119 | **1718** | 143 | 26.4 |
| A, $R_2$ | 132 | 141 | 162 | 149 | 140 | 140 | 137 | 244 | 126 | 129 | 117 | 111 | **1728** | 144 | 23.8 |
| A, $R_B$ | 133 | 141 | 162 | 14 | 170 | 174 | 155 | 134 | 156 | 132 | 119 | 111 | **1737** | 145 | 13.6 |
| A, $F_B$, $R_B$ | 133 | 141 | 162 | 16 | 168 | 174 | 161 | 129 | 149 | 133 | 118 | 111 | **1738** | 145 | 14.1 |
| Work force | - - - - - - - - - - - - - - - - - - - - - - - - - - - - - *person days* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - | | | | | | | | | | | | | | |
| A | 8710 | 2800 | 1335 | 7413 | 11095 | 0 | 22688 | 4590 | 4838 | 3093 | 3675 | 6255 | 76490 | 6374 | 94.3 |
| A, $F_1$ | 4097 | 4712 | 5418 | 6231 | 7166 | 8241 | 9477 | 8055 | 6847 | 5820 | 4947 | 5689 | 76700 | 6392 | 25.2 |
| A, $F_B$ | 4555 | 5011 | 5512 | 6063 | 6669 | 7336 | 8070 | 8191 | 7372 | 6634 | 5971 | 5374 | 76757 | 6396 | 18.5 |
| A, $F_2$ | 5318 | 5584 | 5863 | 6157 | 6464 | 6788 | 7127 | 7483 | 7109 | 6754 | 6416 | 6095 | 77160 | 6430 | 10.2 |
| A, $R_1$ | 7150 | 4360 | 1529 | 7150 | 7150 | 7150 | 7150 | 7150 | 7150 | 7150 | 7150 | 6255 | 76494 | 6374 | 27.2 |
| A, $R_2$ | 5850 | 5850 | 5850 | 5850 | 5850 | 5850 | 6022 | 13233 | 5850 | 5850 | 5850 | 5850 | 77755 | 6480 | 32.8 |
| A, $R_B$ | 5850 | 5850 | 5850 | 5850 | 7150 | 7150 | 7150 | 7150 | 7150 | 6131 | 5850 | 5850 | 76981 | 6415 | 10.2 |
| A, $F_B$, $R_B$ | 5850 | 5850 | 5850 | 6435 | 7079 | 7150 | 7150 | 7150 | 6770 | 6093 | 5850 | 5850 | 77077 | 6423 | 9.3 |
| Productivity | - - - - - - - - - - - - - - - - - - - - - - - - - *person days/hectare* - - - - - - - - - - - - - - - - - - - - - - - - - | | | | | | | | | | | | | | |
| A | 8.30 | 3.11 | 2.70 | 3.92 | 3.33 | --- | 4.37 | 3.99 | 2.60 | 1.89 | 10.50 | 9.55 | | 4.13 | 73.2 |
| A, $F_1$ | 7.61 | 8.90 | 3.48 | 3.64 | 3.03 | 4.15 | 4.24 | 4.24 | 3.99 | 2.35 | 5.47 | 9.51 | | 4.14 | 56.6 |
| A, $F_B$ | 7.76 | 9.12 | 3.36 | 3.68 | 2.97 | 4.10 | 4.48 | 4.22 | 3.65 | 2.77 | 4.47 | 9.48 | | 4.15 | 57.2 |
| A, $F_2$ | 6.17 | 8.08 | 3.44 | 3.69 | 2.94 | 4.01 | 4.58 | 4.36 | 3.43 | 3.74 | 3.95 | 6.62 | | 4.17 | 37.2 |
| A, $R_1$ | 8.02 | 4.12 | 2.91 | 3.86 | 2.98 | 4.07 | 4.35 | 4.58 | 3.43 | 3.25 | 3.78 | 9.55 | | 4.13 | 49.8 |
| A, $R_2$ | 5.18 | 8.26 | 3.43 | 3.67 | 2.96 | 4.24 | 4.17 | 4.37 | 3.48 | 3.11 | 4.46 | 8.69 | | 4.20 | 44.9 |
| A, $R_B$ | 4.93 | 8.26 | 3.43 | 3.67 | 2.98 | 4.07 | 4.49 | 4.41 | 3.44 | 3.17 | 4.60 | 8.82 | | 4.16 | 45.7 |
| A, $F_B$, $R_B$ | 5.18 | 8.26 | 3.43 | 3.70 | 3.02 | 4.07 | 4.22 | 4.71 | 3.40 | 3.21 | 4.50 | 8.11 | | 4.16 | 42.5 |

CV= Coefficient of Variation; A = Annual program; F = Maximum work force fluctuation; R = Range of allowable person days per month.
($F_B$ = 10%, $F_1$ = 15%, $F_2$ = 5%; $R_B$ = 5850 - 7150 person days, $R_1$ >= 5850 person days, $R_2$ <= 7150 person days)

Defining a maximum number of person days to hire per period also improves labor distribution. However, some months of very low activity cannot be avoided, which is enough to generate rotation of labor force. Considering a minimum number of person days solves this problem but tends to concentrate a great amount of work in the most favorable month, an unrealistic solution since it is not possible to hire specialized workers for such a brief period. We obtained a better solution from the perspective of labor stability by restricting both the minimum and maximum number of person days. This situation generated only two levels of activity during the year, with each level corresponding to a boundary of the range.

The simultaneous constraints of range of person days and maximum fluctuation yield a solution that does not exceed critical limits but permits a gradual transition between the two main levels of activity that occur during the season. Also, this formulation is more effective since it produces a smaller variation of person days than any of the restrictions considered independently, without increasing the total cost.

Finally, as expected, higher demands on the conditions of the problem generate better labor stability and a more uniform productivity, but also higher operational costs.

## Effect of the Type of Constraints

We defined several constraint alternatives for the objective function of minimizing total costs. In each formulation, new restrictions were added to the basic condition. The constraint that limits the work force fluctuation was included first, then the one on the range of person days was added, and so on, until the whole set of restrictions that are relevant to the problem of minimizing costs had been added.

Table 5 shows how total costs rise as the constraints of the problem increase, explaining why economic interests oppose those of labor and operative stability. For example, limiting the range of person days has a larger impact on cost than restricting the fluctuation, but it also makes the work force distribution more homogeneous. Adding the restriction of fluctuation to that of the range of person days has a minimum impact on extra costs, but shows an interesting effect on labor stability.

**Table 5.—Effect of type of constraints**

| Analyses | Monthly Results | | | | | | | | | | | | Summary | | Iterations |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Subject to: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total | CV | |
| Cost | - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - *thousand dollars* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - | | | | | | | | | | | | | *percent* | |
| A | 208 | 79 | 39 | 182 | 270 | 0 | 444 | 81 | 104 | 80 | 70 | 119 | **1676** | 87.2 | 1 |
| A, F | 109 | 117 | 153 | 154 | 160 | 178 | 161 | 163 | 154 | 150 | 116 | 103 | **1717** | 17.4 | 259 |
| A, R | 133 | 141 | 162 | 149 | 170 | 174 | 155 | 134 | 156 | 132 | 119 | 111 | **1737** | 13.6 | 194 |
| A, F, R | 133 | 141 | 162 | 161 | 168 | 174 | 161 | 129 | 149 | 133 | 118 | 111 | **1738** | 14.1 | 212 |
| A, S | 124 | 91 | 94 | 125 | 205 | 108 | 231 | 311 | 93 | 108 | 108 | 109 | **1709** | 48.7 | 954 |
| A, S, C | 83 | 112 | 98 | 91 | 196 | 96 | 307 | 294 | 75 | 125 | 141 | 112 | **1729** | 55.3 | 20316 |
| A, S, F | 104 | 125 | 148 | 157 | 165 | 167 | 172 | 165 | 165 | 148 | 125 | 110 | **1753** | 16.3 | 54162 |
| A, S, R | 133 | 141 | 155 | 182 | 172 | 176 | 149 | 137 | 165 | 126 | 123 | 120 | **1780** | 14.6 | 13844 |
| A, S, F, R | 134 | 142 | 157 | 172 | 175 | 190 | 142 | 139 | 171 | 134 | 122 | 127 | **1806** | 14.6 | 16265 |
| A, S, C, F, R | 135 | 132 | 163 | 175 | 175 | 185 | 141 | 139 | 176 | 140 | 120 | 138 | **1818** | 14.4 | 2500000 |
| Work force | - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - *person days* - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - | | | | | | | | | | | | | | |
| A | 8710 | 2800 | 1335 | 7413 | 11095 | 0 | 22688 | 4590 | 4838 | 3093 | 3675 | 6255 | 76490 | 94.3 | |
| A, F | 4555 | 5011 | 5512 | 6063 | 6669 | 7336 | 8070 | 8191 | 7372 | 6634 | 5971 | 5374 | 76757 | 18.5 | |
| A, R | 5850 | 5850 | 5850 | 5850 | 7150 | 7150 | 7150 | 7150 | 7150 | 6131 | 5850 | 5850 | 76981 | 10.2 | |
| A, F, R | 5850 | 5850 | 5850 | 6435 | 7079 | 7150 | 7150 | 7150 | 6770 | 6093 | 5850 | 5850 | 77077 | 9.3 | |
| A, S | 5160 | 3688 | 3323 | 4640 | 8560 | 5123 | 11430 | 16345 | 4363 | 4895 | 5400 | 5480 | 78405 | 58.4 | |
| A, S, C | 3333 | 4290 | 3753 | 3313 | 7993 | 4430 | 14630 | 15550 | 3643 | 5345 | 6895 | 5590 | 78763 | 64.6 | |
| A, S, F | 4602 | 5062 | 5569 | 6125 | 6738 | 7412 | 8153 | 8429 | 7586 | 6827 | 6145 | 5530 | 78177 | 18.7 | |
| A, S, R | 5850 | 5850 | 5850 | 7150 | 7150 | 7150 | 7150 | 7150 | 7150 | 5999 | 5850 | 5850 | | 10.2 | |
| A, S, F, R | 5850 | 5850 | 6077 | 6685 | 7150 | 7150 | 7150 | 7150 | 7150 | 6435 | 5850 | 5850 | | 9.2 | |
| A, S, C, F, R | 5850 | 5850 | 6149 | 6764 | 7150 | 7150 | 7150 | 7150 | 7150 | 6435 | 5850 | 5850 | | 9.2 | |

CV= Coefficient of Variation; A = Annual program; F = Maximum workforce fluctuation; R = Range of allowable person days per month; S = Minimum and maximum size per activity at each tree-farm; C = Continuity condition.

Simultaneously considering the constraints of minimum and maximum area per month per task and the condition of continuity avoids a series of operative problems but increases the total cost by 3.2 percent. When adding range and fluctuation considerations to such constraints, total cost increases 8.5 percent with regards to the basic model of minimizing costs (from thousand-$ 1676 to 1818), while the variation coefficient of the work force hired during the year decreases from 94.3 to 9.2 percent.

In terms of the computer processing time required to solve each model, the results indicate that when the complete set of constraints is used, a combinatorial problem that is difficult to solve results, due mainly to a conflict between the continuity condition and the restrictions related to work force distribution (range and fluctuation). Under this situation it was not possible to find an optimum solution in less than 2.5 million iterations, equivalent to approximately 12 hours of computer processing. However, if a tolerance of 3.5 percent were considered with regards to the theoretical limit, a good solution would have been found in nearly 30,000 iterations. All the other formulations of the problem had a solution time of less than 20 minutes.

## Trade Off Analysis: Cost-Labor Stability

The problem of minimizing the work force variance was solved taking into consideration different levels for the maximum total cost allowed. Table 6 shows the main results of this analysis and Figure 2 presents the curve based on these results.

**Table 6.—Trade-off analysis when trying to minimize work force variance: cost-labor stability**

| Analyses | Total | Monthly Results | | | | | |
|---|---|---|---|---|---|---|---|
| Subject to: | | Mean | Median | Minimum | Maximum | Range | CV |
| Cost | | - - - - - - - - | - - - *thousand dollars* | - - - | - - - - - - - - | | *percent* |
| A | 1821 | 152 | 147 | 121 | 180 | 58 | 11.9 |
| A, $P_1$ | 1790 | 149 | 147 | 123 | 180 | 58 | 12.7 |
| A, $P_2$ | 1760 | 147 | 145 | 115 | 178 | 62 | 12.8 |
| A, $P_3$ | 1730 | 144 | 142 | 113 | 176 | 63 | 12.8 |
| A, $P_4$ | 1700 | 142 | 132 | 103 | 193 | 90 | 22.6 |
| A, $P_B$ | 1676 | 140 | 111 | 63 | 272 | 209 | 50.7 |
| Work force | | - - - - - - - - | - - - *person days* | - - - | - - - - - - - - | | |
| A | 78208 | 6517 | 6517 | 6486 | 6551 | 65 | **0.5** |
| A, $P_1$ | 77861 | 6488 | 6482 | 6411 | 6579 | 168 | **1.2** |
| A, $P_2$ | 77003 | 6417 | 6396 | 6190 | 6657 | 467 | **3.1** |
| A, $P_3$ | 76917 | 6410 | 6317 | 5360 | 7427 | 2067 | **11.4** |
| A, $P_4$ | 76586 | 6382 | 6324 | 4135 | 8509 | 4374 | **24.8** |
| A, $P_B$ | 76875 | 6406 | 4355 | 2333 | 13604 | 11271 | **59.7** |

CV= Coefficient of Variation; A = Annual program; P = Maximum budget. ($P_B$ = 1676 thousand-$, $P_1$ = 1790 thousand-$, $P_2$ = 1760 thousand-$, $P_3$ =1730 thousand-$, $P_4$ = 1700 thousand-$)

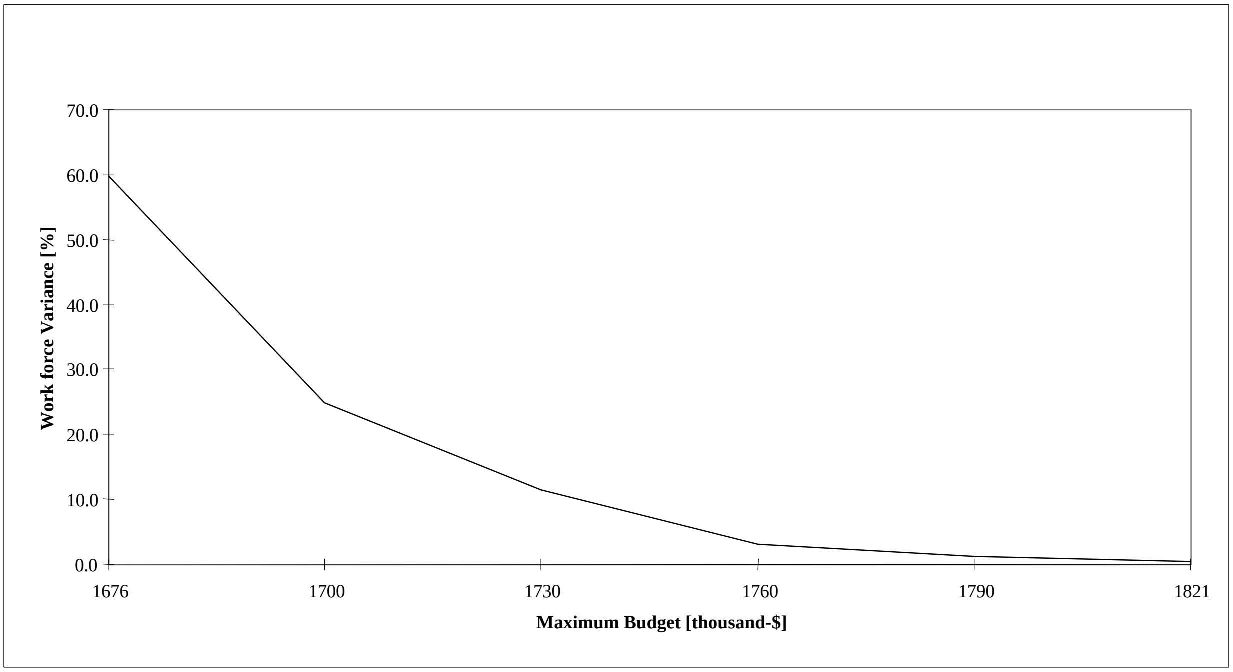


**Figure 2.—Trade-off curve: cost-labor stability.**

This formulation of the problem defines the best labor stability one can possibly achieve without exceeding a certain budget for the execution of all the tasks. For example, a variation of 1.2 percent is obtained for a budget of $ 1.79 million. This is less than the variation of 6.6 percent for the solution of the MMW model that has the same cost (Table 3). In the case of minimizing the total cost subject to range and fluctuation constraints, the variation in person days is 9.2 percent with a cost of $ 1.74 million (Table 4). With the same budget, a coefficient of variation of 8.5 percent takes place when the work force variance is minimized. The variation increases to 11.4 percent as the maximum total cost allowed is lowered to $ 1.73 million.

The curve that results from the different budget levels corresponds to the most efficient transaction points in terms of the increase in cost required to obtain a better labor stability, based on a given situation. In the case analyzed in this study, the curve shows a relatively favorable cost-stability transaction with regards to labor, up to a cost of $ 1.73 million. From that point on, better stability is only achieved with considerable increases in the total cost.

## Dominant Solutions

We analyzed two types of problems for the objective functions for minimizing variance and MMW. The first one only considered the execution of the program of silvicultural tasks. The second adds restrictions for the minimum and maximum area of a task per month and tree farm. Both problems included an additional constraint that limited the total cost allowed for the solution, using the value obtained upon minimizing the cost subject to the same type of restrictions as a reference budget.

Table 7 indicates that it is possible to improve labor stability without increasing operative costs. In models without restrictions for minimum and maximum area, the variation coefficient decreases from 94.3 to 67.0 percent, for the objective function minimizing the MMW, and to 59.7 percent, for that of minimizing the variance. For the models that include area restrictions, profits—in terms of labor stability—are less since these restrictions themselves have a regulating effect when the total cost is minimized. However, since the operative budget is larger, there is more latitude for the objective functions of minimizing variance and MMW to find a more stable labor solution, with a drop in the variation coefficients from 59.7 to 48.5 percent, and from 67.0 to 53.5 percent, respectively.

**Table 7.—Generation of dominant solutions**

| Analyses | | Total | Average Monthly Results | | | | | |
|---|---|---|---|---|---|---|---|---|
| Minimized item | Subject to | | Mean | Median | Minimum | Maximum | Range | CV |
| Cost | | | - - - - - - - - - - - - - - - - *thousand dollars* | - - - - - - - - - - - - - - - - | | | | *percent* |
| Total Cost | A | **1676** | 140 | 92 | 0 | 444 | 444 | 87.2 |
| MMW | A, $P_B$ | 1676 | 140 | 124 | 39 | 278 | 239 | 60.9 |
| Work force Variance | A, $P_B$ | 1676 | 140 | 111 | 63 | 272 | 209 | 50.7 |
| Total Cost | A, S | 1709 | 142 | 109 | 91 | 311 | 220 | 48.7 |
| MMW | A, S, $P_S$ | 1709 | 142 | 117 | 62 | 255 | 193 | 46.5 |
| Work force Variance | A, S, $P_S$ | 1709 | 142 | 114 | 94 | 258 | 164 | 40.1 |
| Work force | | | - - - - - - - - - - - - - - - - - - *person days* | - - - - - - - - - - - - - - - - - - | | | | |
| Total Cost | A | 76490 | 6374 | 4714 | 0 | 22688 | 22688 | 94.3 |
| MMW | A, $P_B$ | 76903 | 6409 | 6228 | 1335 | **13604** | 12269 | 67.0 |
| Work force Variance | A, $P_B$ | 76875 | 6406 | 4355 | 2333 | 13604 | 11271 | **59.7** |
| Total Cost | A, S | 78405 | 6534 | 5141 | 3323 | 16345 | 13023 | 58.4 |
| MMW | A, S, $P_S$ | 77863 | 6489 | 5385 | 2148 | **12869** | 10721 | 53.5 |
| Work force Variance | A, S, $P_S$ | 78287 | 6524 | 5121 | 3323 | 12863 | 9540 | **48.5** |

CV= Coefficient of Variation; A = Annual program; S = Minimum and maximum size per activity at each tree-farm; P = Maximum budget. ($P_B$ = 1676 thousand-$, $P_S$ = 1709 thousand-$)

Given the same budget limit, the best answer, in terms of labor stability, is achieved with the minimum variance model. However, it should be kept in mind that this objective function is a quadratic form, which becomes a difficult problem to solve with integer variables. The objective function of minimizing MMW, on the other hand, is a min-max type model of linear nature, that enables obtaining solutions in considerably less time, more so when the size of the problem increases.

The optimization procedure employed generates dominant solutions in the sense that they tend to maximize labor stability without increasing the total cost. By solving the problem of minimizing total costs, the most economically convenient solution is obtained. When employing the resulting value as a budget restriction in models oriented to level out the work force distribution, it is possible to achieve the best final solution from the perspective of combining two conflicting interests.

## CONCLUSIONS

The models mentioned in the different analyses that have been carried out highlight how the problem of scheduling silvicultural tasks can be formulated in multiple ways according to the objectives and priorities considered at the moment of planning the activities.

The objective function of minimizing total costs prioritizes economic interests. Nevertheless, it is possible to significantly improve labor stability by restricting the range of person days to be hired and their maximum fluctuation over time. The best results are achieved combining both restrictions, since the second one only controls the variation between consecutive months, but does not avoid the fluctuations accumulated throughout the season. This situation is corrected by restricting, in absolute terms, the minimum and maximum work force to be hired in any period.

The best work force distribution in the season is obtained through the objective function that minimizes the person days variance, although the model employs a poor allocation of productivity that conveys higher operational costs in order to achieve this distribution. The objective function of minimizing MMW tends to standardize the distribution of person days as well. It has the advantage of being a linear model that requires a shorter solution time. Also, it tends to generate less demand of labor force and, consequently, a lower total cost. However, this objective function becomes indifferent to the distribution of person days through time once it is not possible to reduce the work force required for a certain month. Therefore, it is not very effective when activity levels vary much during the year.

The results of this study show how economic interests oppose those of labor and operative stability. Consequently, it is necessary to compare several formulations in order to achieve a solution that harmonizes the different interests of the company. In particular, the generation of dominant solutions presents a good option for overcoming conflicting objectives by allowing an improvement in labor stability without increasing total costs.

In general, through the type of analysis carried out in this study, one can determine the effect that the objective function and the different constraints have in the solution of the problem. Also, it enables the professional entrusted with the planning of silvicultural tasks to evaluate the additional costs that are implied to satisfy the diverse requirements of the company. This study highlights the importance of applying mathematical programming techniques for efficiently planning silvicultural tasks at an operative level.

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