HARVEST PLANNING WITH UNCERTAIN FUTURE PRICES AND DEMANDS

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ABSTRACT. —Harvest planning involves making harvesting decisions over multiple time periods. Usually the demands and prices for a set of log products are known for the first period but are uncertain in later periods. The typical approach to this situation is to run a set of models to explore the effect of different price scenarios. This, however, explores only a small subset of all possible future outcomes and does not compute a plan that hedges against all scenarios. Stochastic linear programming models can do this but tend to be very large and therefore difficult to solve. This paper describes a stochastic forestry modeling system and discusses the use of decomposition to solve such models.

INTRODUCTION

Most forestry decision-making occurs under risk or uncertainty. Forestry's long production cycle exacerbates this problem. Uncertainties exist in the prediction of future growth and yields, the effect of outside influences (e.g., the risk of fire), and the marketplace. These predictions are at best guesses that are subject to error.

The problem examined in this paper is harvest planning and log allocation with uncertain future demand and prices. Spread throughout the forest estate is a number of candidate stands for harvest over a number of time periods. The yields from these stands are represented as different log product types that are sold to potential customers. Each customer specifies a set of demands. A demand represents a constraint on the volume of a set of log types for which an at-mill price will be paid. In a deterministic model, demands and prices are specified in advance for all periods. While the short-term prices and demands may be known, this is seldom the case for future time periods.

Traditionally some form of sensitivity analysis is performed to quantify likely scenarios. While this practice is useful, it explores only a small subset of all possible futures. A deterministic model based upon a single scenario also contains an implicit assumption of perfect information; i.e., the demands and prices for all periods are known in advance.

In short-term harvest planning, a number of stands are scheduled for harvesting. One must then decide when it is economically desirable to harvest each stand. This decision is dependent not only on the current prices and demands for log products during that period, but also on future prices and what the future demand will likely be. It is likely that a shrewd forest manager will hedge against the future in the event of higher prices, lower prices, or changing demand. When the outcome of a decision becomes known, a recourse, or corrective action, can be made.

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STOCHASTIC LINEAR PROGRAMS WITH RECOURSE

Recourse problems are staged problems where the decision maker alternates decisions with realizations of stochastic data. Decisions are made for the current stage given assumptions of the probability of future events, or outcomes. After these decisions are made, the actual occurrences of some stochastic events become known. Based on the previous decisions and the new information obtained, an optimal corrective action is taken as the next stage is considered. This process closely approximates real-life dynamic decision making. The solution of the Stochastic Programming with Recourse (SPR) problem gives the recommended strategy for every defined scenario.

Few papers have considered multistage recourse as forest estate models are typically large and do not incorporate uncertainty. Hoganson and Rose (1987) developed a decomposition technique for obtaining approximate solutions to stochastic forest planning problems. They specified complicating constraints as penalty terms in the objective function by using estimates of the dual variables as penalty weights. They formulated the problem as a model with two stages—one representing the long run and the other representing the short run. The objective was to minimize the cost of producing the desired forest outputs where the future forest output levels were uncertain.

Gassman (1989) developed a multistage recourse model for finding logging levels to maximize value. The risk of forest fires or other environmental hazards, which might destroy a random fraction of the existing forest, was uncertain. In his paper, he made the following conclusions:

- stochastic models lead to more conservative harvesting policies than deterministic ones,
- the value of information increases as the discount factor increases, and
- the solution to the mean value problem (MVP), where expected values are substituted for stochastic elements, becomes more unrealistic with increasing discount rates.

Boychuk and Martell (1996) used multistage stochastic programming to investigate the risk of fire. Two outcomes were possible in each of the first four periods, representing a high and a low fire loss, after which the expected fire loss was assumed. They compared their SPR model with the MVP, where the fire loss in each period was the expected value. They concluded that the MVP consistently overharvests, risking the possibility of a future shortfall under some scenarios.

The Equivalent Deterministic Problem (EDP) is the linear program that describes the SPR problem. The number of possible scenarios depends exponentially upon the number of decision stages, although a stage may represent more than one time period. The EDP is roughly the size of the original problem multiplied by the number of possible scenarios. Therefore, stochastic programs can very easily become too large to solve. For this reason, SPR models are as an intuitive approach to uncertainty as long as the number of stages is small.

This paper describes the creation of the EDP in the FOLPI forest estate modeling system. It discusses the regular structure of these problems as well as possible decomposition techniques to solve them. It also describes a model based on Stochastic Benders' Decomposition (SBD) and compares the results with EDP and MVP.

BUILDING A DETERMINISTIC EQUIVALENT PROBLEM WITH FOLPI

FOLPI is a linear-programming-based, forest estate modeling system that has been used in New Zealand for over 10 years for forest management planning, log allocation, and forest valuation. It corresponds to a Model B formulation (García 1990), which is an extension of the Model II formulation originally proposed by Johnson and Scheurman (1977). García (1990) describes the FOLPI formulation. Manley and Threadgill (1991) discuss the use of log allocation by FOLPI for the purpose of forest valuation. FOLPI has had considerable use in short-term planning where it determines harvesting decisions for individual stands (Manley 1994). Recent use of FOLPI has incorporated process modeling where processing plant output is included within the modeling system.

There is an analogy between the concept of future price scenarios and the use of log allocation in FOLPI to optimize and satisfy the demands of different products from a set of origins to a set of destinations. Each future

scenario may be represented by a set of destinations, demands, and prices. Only one scenario may come true, so each scenario has an estimated probability of occurring.

A deterministic equivalent formulation of a SPR problem in FOLPI can be built by assigning each scenario its own unique set of decision variables. An additional required set of constraints forces harvesting to be the same for scenarios with identical outcomes up until and including the period in question. These nonanticipative constraints are of the form:

$$y_{tij1} = y_{tij2} = \ldots = y_{tijn}$$

where y_{tijq} represents the area cut in period *t* of crop type *i* in age class *j* for scenario *q*. Scenario number *n* varies by problem and stage depending upon the information available for the current stage and the possible future outcomes.

EXAMPLE

It is assumed that the harvest planner is in a learn-demand-then-produce situation. For a given period *t*, the planner knows the exact demand for that period as well as the values for previous demands. The planner may conceivably be in a produce-then-learn-demand situation, where the setting of the harvesting level for period *t* must be decided before the demand is known. While the planner may not really know the exact demand at the start of the period, this information becomes available during the period and the planner can usually readjust harvesting levels if required.

There are three possible price outcomes in this example: high, medium, and low. Harvesting produces radiata pine logs of varying quality: pruned logs, A- and J-grade sawlogs, poor quality sawlogs, and pulp logs. Table 1 lists each possible price outcome for these logs.

Table 1. —Price List by Product and Outcome (New Zealand dollars per cubic meter)

Product	High Prices	Medium Prices	Low Prices	Average
RAD PA	$220.00	$200.00	$160.00	$193.33
RAD PB	$150.00	$130.00	$100.00	$126.67
RAD AG	$130.00	$110.00	$90.00	$110.00
RAD JG	$110.00	$100.00	$80.00	$96.67
RAD RT	$70.00	$55.00	$45.00	$56.67
RAD PU	$40.00	$35.00	$25.00	$33.33

RAD = radiata pine; PA = pruned logs; PB = pruned logs; AG = A-grade sawlogs; JG = J-grade sawlogs; RT = poor quality sawlogs; PU = pulp logs.

A set of possible future scenarios may be represented as an event tree. This is a diagram that branches off for each possible outcome at each stage. A three-stage (one stage equals one period equals 1 year) model might have an event tree like the one in Figure 1 if the known prices for the current period (year 1) were the medium prices.

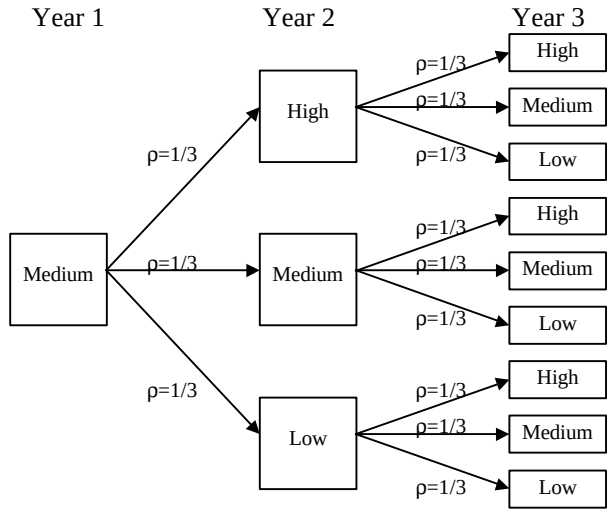

Figure 1. —Event Tree.

There is an equal probability of any outcome occurring in this example. This need not be the case; the sole requirement is that the conditional probability of all possible future outcomes sum to 1.

A decision has to be made as to how much of the remaining area is to be harvested in the final period. To make that decision, some value must be placed on the nonharvested area at the end of the model. Calculating this boundary value is common practice in forestry linear programming models and is used to minimize end effects when overharvesting of the forest occurs towards the end of the planning horizon. In this case, the boundary value is calculated from the land expectation value, the expected discounted net cash flow assuming average prices over infinite periods when there are no yield regulation constraints.

It is a credit to the flexibility of the FOLPI modeling system that a complex, stochastic, equivalent deterministic model can be created without much difficulty. The one drawback to using FOLPI in this way is that each scenario must carry its own unique set of harvesting and wood-flow variables. Many of these variables can be removed because of nonanticipative constraints. A model based on the event tree in Figure 1 would require the values of harvest variables to be the same for all scenarios in year 1:

$$y_{1ij1} = y_{1ij2} = y_{1ij3} = y_{1ij4} = y_{1ij5} = y_{1ij6} = y_{1ij7} = y_{1ij8} = y_{1ij9} \qquad \forall i, j$$

Meanwhile, in period 2, the harvest variables would be the same for all scenarios that shared the same outcome for period 2:

$$y_{2ij1} = y_{2ij2} = y_{2ij3} \qquad y_{2ij4} = y_{2ij5} = y_{2ij6} \qquad y_{2ij7} = y_{2ij8} = y_{2ij9} \qquad \forall i, j$$

Those variables that are assumed to share equality can be merged into a single variable.

DECOMPOSITION

Although equivalent deterministic programs are large, they inevitably have a regular structure that can be exploited. The SPR model has a definite staircase structure where the nonanticipative constraints bind across stages (Fig. 2).

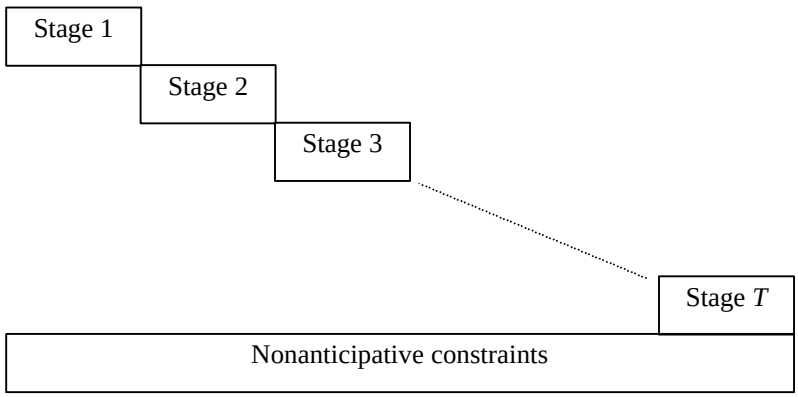

Figure 2.—Staircase structure of the equivalent deterministic model.

A staircase structure by scenario is also possible by reordering of the constraints and variables.

STOCHASTIC BENDER'S DECOMPOSITION

Considerable promise has been shown for SBD, which has also been referred to as the L-shaped Decomposition Method (e.g. Kall and Wallace 1994). Each subproblem represents a possible outcome for a stage, i.e., a single node on the event tree. SBD is resource-driven: the ancestor subproblem explicitly allocates the resources and the subproblems find the best solutions. Variables are split into "hard" and "easy" sets. Due to the tightly coupled nature of the FOLPI structural constraints, it is not possible to separate the variables. The solution is to introduce state variables that describe the area in each period, crop type, and age class. This is a Model A (García 1990) "state space" formulation that considerably increases the number of variables but simplifies the form of the structural constraints. Reformulation of the FOLPI structural constraints makes the constraints as period independent as possible.

Let s_{tij} be the area at the start of period *t* in crop type *I* and age class *j*. SBD is based on the observation that given values for s_{tij} from the solution of the ancestor problem in period *t*-1, the decision problem can be written in the form:

$$\max a_t\left(\sum_{i,j} C_{tij}s_{tij} + \sum_{p,f,d} R_{tpfd}u_{tpfd}\right) + Q_{t+1}(s)$$

subject to

(1) $\sum_{,j} y_{tij} = \sum_k r_{tik}$ $\quad \forall i$

(2) $s_{1ij} - \sum_k z_{ijk} + \sum_k z_{kji} = A_{ij}$ $\quad \forall i, j \quad$ if $t = 1$

(3) $s_{t+1,i,1} = \sum_k r_{tki}$ $\quad \forall i$

(4) $y_{tij} + s_{t+1,i,j+1} = s_{tij}$ $\quad \forall i, j$

(5) User defined constraints

(6) $\sum_d u_{tpfd} = \sum_{\substack{j \\ i \in f}} V_{ijp} y_{tij}$ $\quad \forall p, f$

(7) $\sum_{\substack{f \\ p \in l}} u_{tpfd} = D_{tdl}$ $\quad \forall d, l$

(8) $y_{tij}, s_{tij}, r_{tik}, z_{ijk} \geq 0$ $\forall i, j, k$

(9) $u_{tpfd} \geq 0$ $\forall p, f, d$

where:

$Q_{t+1}(s)$ = a concave function representing the present expected value of all possible future scenarios,

a_t = the discount rate in period *t*,

y_{tij} = area harvested of crop type *i* in age class *j*,

r_{tik} = area replanted from crop type *i* to crop type *k*,

z_{ijk} = area in age class *j* transferred from crop type *i* to crop type *k* at the start of the planning period,

u_{tpfd} = the wood flow of log product type *p*, from forest *f* to destination *d*,

A_{ij} = initial area in crop type *i*, age class *j*,

C_{tij} = deterministic revenues and costs occurring during the rotation,

R_{tpfd} = the revenue gained from selling one unit volume of log product *p* from origin *f* to destination *d*,

D_{tdl} = the demand at destination *d* for *l*, a set of log product types *p*, and

V_{ijp} = the volume of log product type *p* produced by clear-felling one unit of area of crop type *i*, age class *j*.

Translated into words, these constraints state the following:

1. the area harvested must be replanted,
2. the area in a crop type and age class at the start of period 1 equals the area initially available, less the amount transferred out, plus the amount transferred into that crop type and age class,
3. the area in a crop type in age class one at the start of the subsequent period equals the amount replanted into that crop type during the current period,
4. the area for a given crop type and age class at the start of the subsequent period equals the amount of area in the current period and age class less the amount that was harvested during the period,
5. constraints on the forest resource representing restrictions placed on the resource by outside factors,
6. the wood flow to all destinations or customers for a forest and log product equals the sum of the volume of that log type cut from all crop types in that forest, and
7. the definition of the destination demands in the current outcome.

Constraint sets 3, 4, and possibly 5 are coupling constraints which reference solely the subsequent stage. This is a necessary restriction to simplify the SBD solution process; additional constraints defined by the user (5) cannot violate this restriction. This requirement is not too restrictive as user constraints usually set a bound for a period or constrain a resource between consecutive periods. The nonanticipative constraints are implicit in the decomposition and are unnecessary. The ancestor problem passes values for the s_{tij} variables to the current subproblem. These values are used as the right hand side in constraint set 4.

The function $Q_{t+1}(s)$ is approximated by adding an unrestricted variable q in the objective function. This variable is constrained by the addition of Benders' cut constraints, so that q gives an upper bound estimate on the value of the subproblems for the stage.

It is easiest to explain the form of Benders' cut by considering a problem with only one state variable s (i.e., only one crop type and one age class) representing the amount of area passed down to a single subproblem beneath it. In this case, a revenue curve could be drawn representing the revenue earned (q) versus the amount of area (s) passed down to the subproblem (Fig. 3).

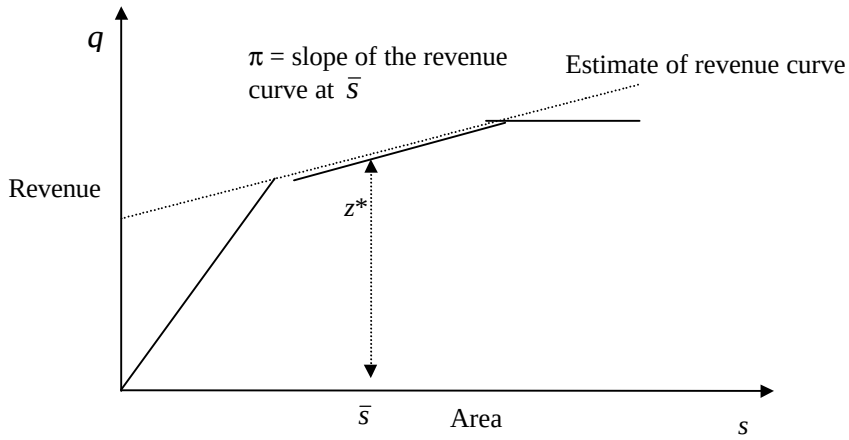

Figure 3. —Revenue curve for state variable s.

The actual form of this revenue curve is unknown. The only information available upon solution of the subproblem is

- $\bar{s}$, the value of s passed down to the subproblem,
- z^*, the objective function value of the subproblem, and
- p, the marginal revenue at $\bar{s}$, i.e., the dual of the constraint in the subproblem with right hand side $\bar{s}$.

Given solely this information, the best estimate we can derive for q is a straight line that gives a revenue of z^* at $\bar{s}$ and has slope of p. An estimate for q is therefore $ps + (z^* - p\bar{s})$. Assuming that the revenue function over the allowable clear-felling ages is concave, our estimate for q is an upper bound. As the subproblem is solved repeatedly with differing values for $\bar{s}$, the estimator for q approximates the revenue curve more closely.

The upper bound gives an estimate for the value of subsequent periods by defining q in terms of the areas passed down to the subproblems in the state variables $s_{t+1,i,j}$. Summing over all state variables and all possible subproblem outcomes x weighted by their probabilities gives the generalized Benders' cut:

$$q \le \sum_x r_x \left(z_x^* + \sum_{i,j} p_{xij} s_{t+1,i,j} - p_{xij} \bar{s}_{t+1,i,j} \right)$$

In summary, each problem passes down the area/age class distribution as right hand side values $\bar{s}_{t+1,i,j}$ to its descendant subproblems. Upon solution, each subproblem contributes towards the formulation of an additional Benders' cut constraint that constrains q with respect to the state variables $s_{t+1,i,j}$, giving an estimate of future value. After being solved again, the problem obtains a new value for q, a new cutting plan for the period, and changes the initial area/age class distribution for the subproblems. Figure 4 shows the relationship between SBD subproblems.

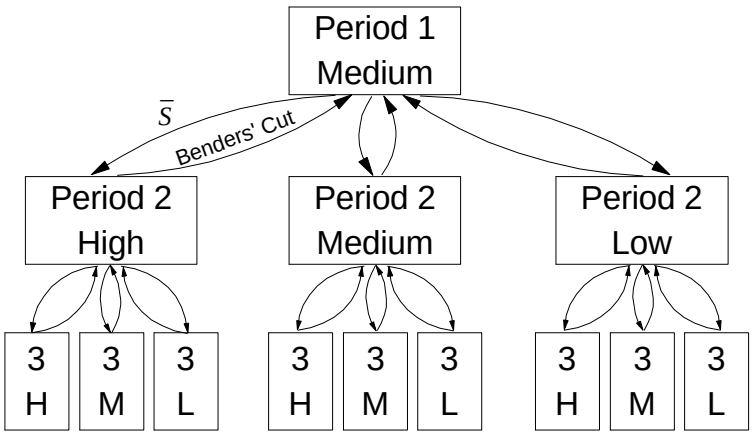

Figure 4. —Relationship between SBD subproblems.

Subproblems may be solved in any order providing the ancestor subproblem has been solved at least once. The algorithm terminates when the objective function value (including q) for the period 1 subproblem is within a tolerance of the current solution (excluding q) plus the nested sum of the probability-weighted solution values for all descendant subproblems. Initial harvesting plans for a period will be aggressive, as no consideration is given towards the potential future earnings of future periods. As Benders' cuts are added, the amount harvested for an outcome will tend towards a level that weighs harvesting at current prices against the opportunity of an additional year's growth, the effect of discounting, and the uncertainty of future prices. The optimal harvesting strategy will inevitably harvest as much as possible when prices are high, and as little as possible when prices are low.

RESULTS

The implementation of the SBD model was developed with the Watcom C++ compiler and the CPLEX Callable Library. Each solution was compared with the MVP and the EDP generated by FOLPI as Mathematical Programming System format files and solved by the CPLEX Simplex solver. Table 2 shows the results of this comparison.

Two forest resource descriptions were used:

- a one crop type model representing 300 hectares (ha) of radiata pine, which assumes that the entire resource grows and is managed the same way,
- a forest resource of 9,406 ha described by twenty crop types, representing a heterogeneous radiata pine forest with differing growth and silviculture.

Three outcomes were possible in each period using the prices from Table 1. An upper and lower bound was placed upon demand while a boundary value ensured that overcutting of the resource did not take place. The one crop type model could not harvest more than 50,000 cubic meters (m^3), while the twenty crop type model was required to supply between 100,000 m^3 and 500,000 m^3. A severe penalty ($500 per m^3) was placed on unsatisfied demand, so solutions feasible for all possible future scenarios were always found. The objective function value represents the expected future revenue as no costs were assumed in the model. A discount rate of 10 percent was used. This is high by international standards but not unreasonable for the New Zealand situation.

Table 2. —Comparison between Stochastic Benders' Decomposition, Mean Value Problem, and the Equivalent Deterministic Problem

Solution Method	Stages	Scenarios	Sub-problems	Crop types	Constraints	Variables	Non-Zeros	Solution Time	Objective Function Value
								min:sec	*NZ$*
MVP	2	3		1	9	15	42	0:01	8,245,539
EDP	2	3		1	154	219	624	0:03	8,452,583
SBD	2	3	1+3	1	93	96	131	0:04	8,452,541
MVP	3	9		1	10	19	52	0:01	8,263,365
EDP	3	9		1	428	409	1,823	0:05	8,643,842
SBD	3	9	1+3+9	1	93	96	131	0:06	8,643,827
MVP	2	3		20	129	278	859	0:09	256,113,552
EDP	2	3		20	3,314	5,778	11,541	1:36	256,809,888
SBD	2	3	1+3	20	1,748	1,775	2,190	1:40	256,809,394
MVP	3	9		20	149	356	1,169	0:10	254,577,313
EDP	3	9		20	5,957	10,187	23,717	3:56	271,486,944
SBD	3	9	1+3+9	20	1,668	1,699	2,068	4:27	271,486,594
MVP	4	27		20	169	437	1,496	0:08	251,667,659
EDP	4	27		20	22,212	13,040	65,069	5:52	253,969,772
SBD	4	27	1+3+9+27	20	1,729	1,775	2,170	8:20	253,778,114

NZ$ = New Zealand dollars; MVP = mean value problem; EDP = equivalent deterministic problem; SBD = Stochastic Benders' Decomposition.

All except the four-stage models were solved on a 90 megahertz (MHz) Pentium with 32 megabytes (MB) of RAM running Windows NT. The four-stage EDP model used virtual memory that slowed solution time considerably. To ensure that the comparative solution times would not be skewed by disk paging, the four stage problems were solved on a Windows NT 200 MHz Pentium with 96 MB of RAM. The reasonable solution times indicate stochastic models represented by large forest resource descriptions are possible. The equivalent deterministic and stochastic Benders' models should yield the same objective function value: the solutions are identical, and the difference is due to rounding in the calculation of the boundary value.

The difference between the MVP and recourse models represents the benefit of hedging or the value of the stochastic solution. Although these models are simplistic, they nevertheless illustrate that the use of expected values, in an environment of volatile prices and flexible supply contracts, will underestimate the value of the resource.

The SBD models took longer to find an optimal solution than their EDP counterparts. This was expected and is a common complaint of decomposition algorithms. There is scope for improvement for solution times. Currently the system uses shuffle pass (as described by Jacobs et al. 1995) where subproblems are solved in their entirety before a Benders' cut is added to the current problem. This is intuitively less efficient than fast pass, where a single cut is made before control returns to the ancestor program, as shuffle pass concentrates its search in a possibly unprofitable area of the feasible solution space. Initial comparisons between the two methods indicate that the fast pass is only slightly faster. A possible approach for improving solution time is for every subproblem to generate its own unique Benders' cut for its predecessor rather than an aggregate Benders' cut over all subproblems. Another possible investigative area when subproblems become difficult to solve is the use of inexact cuts (Philpott et al. 1997) where an interior point method is used to find good, but not necessarily optimal, feasible solutions that will then be used to generate a Benders' cut.

The real advantage of SBD lies with its tractability. Very large models that can not be solved as an equivalent deterministic model can be solved by SBD as only a single subproblem needs to be in memory at one time. The problem of solution time becomes less of an issue with the increased use of parallel and distributed computing. Subproblems can be assigned to separate processors and solved independently since the data flow between processors is small. Very large problems then become solvable within a short period of time.

CONCLUSIONS

SPR's become considerably harder to solve as the number of stages increase due to the exponential nature of the problem. On the other hand, computing power has also increased at a near exponential rate and new algorithms such as SBD have become available to exploit the special structure of stochastic programs.

A model has been developed that uses SBD to solve SPR models for harvest scheduling and log allocation. While this model does not yet find the optimal solution as quickly as the equivalent deterministic model, its tractability lends advantages when multiple processors are available or models become too large to fit into available memory.

The next stage is to apply this model to a practical New Zealand forestry situation. It seems likely that a harvesting plan from an SPR model will derive more value from the resource than a deterministic plan with expected values. Realistic demands and prices will be required before comparing the MVP with a SPR. A key part of this exercise will be the generation of scenarios to be evaluated. These scenarios will represent potential future markets described as a set of possible demands and prices by log product type.

The modeling system described in this paper could also be used to measure the effect of catastrophic risk, since a catastrophe is a possible future outcome with a small probability of occurrence.

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