ISSUES IN THE ESTIMATION OF DYNAMIC MICROECONOMIC MODELS OF FOREST USE

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ABSTRACT. —To date the literature on the microeconomics of forest management has had a normative focus: what forest managers *should* do to maximize a particular objective function. This paper has a positive focus, with particular emphasis on the estimation of dynamic structural models of forest use and their reduced form counterparts.

INTRODUCTION

In the analysis of dynamic decision problems, the vast majority of the literature has focused on the *normative* aspects of such problems: what resource managers *should* do to maximize a particular objective function. So, for instance, the Faustmann model and its many derivatives address the question of when a timber stand *should* be harvested or thinned. There are two reasons for research on the normative questions of dynamic resource management. The first is purely pedagogical: the matter of how to allocate resources over time is a difficult one, and mathematical models of the decision process provide insights to the underlying economics. The second is practical: resource managers and policymakers wish to know how a resource should be managed over time to maximize a particular objective function, such as expected net revenues.

Rarely have resource economists employed their models in the pursuit of the *positive* question; that is, what decision problem do resource managers actually solve? Usually in statistical inference concerning the actual decisions of resource managers and users, dynamic models are treated rather allegorically, with the analyst turning to them for inspiration—to determine, for instance, what variables to use in a linear regression—but never estimating the models directly. The primary explanation for this shortcoming in the literature is that such estimation is computationally very difficult. It usually requires a large amount of computer processing time, and perhaps more importantly, it requires a substantial amount of difficult programming by the analyst. Nonetheless, in the economics literature the estimation of dynamic decision processes has continued to grow rapidly. Most of this literature has concerned dynamic decisions that are dichotomous. Examples related to agricultural and resource economics include the timber harvest decision (Provencher 1995a; 1995b); the decision to take a recreational fishing trip (Provencher and Bishop 1997); and the decision to cull a cow from a dairy herd (Miranda and Schnitkey 1995).

This paper is motivated by the perspective that in the near term the greatest potential for economics to contribute to an understanding of applied dynamic resource management will be positive rather than normative. Over the past twenty years economics has made a substantial contribution to the theory of dynamic resource management; its pedagogical contribution will remain substantial. But its normative contribution to applied dynamic resources management has been and will continue to be minor, for the simple reason that the "curse of dimensionality" prevents the development of decision models that fully reflect the complexity of the social, biological, and physical processes characterizing the decision environment.[2] Without such complexity, resource managers are likely to dismiss the models as irrelevant academic exercises. In dynamic analysis, models with six to eight state variables

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[2] The curse of dimensionality refers to the phenomenon that the computational complexity of a solution algorithm —the number of executable statements necessary to solve a problem—increases exponentially with the dimensionality of the problem. In dynamic programming, references to the curse usually concern the dimension of the state space, though it applies to the dimensionality of the decision space as well.

are computationally very large; yet in my experience resource managers judge such models to be wholly inadequate as decision-making tools.

As discussed below, the curse of dimensionality is even *more* of an issue in the estimation of dynamic decision problems than in the normative solution of such problems, suggesting that if anything, to be computationally tractable a model used in estimation of a dynamic decision problem must be *simpler* than its normative counterpart. Yet the very feature of normative dynamic analyses that leaves them unattractive to resource managers is their simplicity. How then to reconcile the need for tractable models to the contention that economics has much to contribute to the positive analysis of dynamic resource management? The answer is forthcoming from almost any good econometric analysis of behavior. Show such an analysis to the "agent" represented by the model—a consumer, producer, or bureaucrat—and the agent would surely argue against the simplicity of the model. Yet economists continue to use these models, supposedly because they do reasonably well in predicting behavior. Indeed, it appears that the literature on the economics of dynamic resource use is unusual if not unique in its normative emphasis. So the justification for using relatively simple dynamic models to explain the behavior of resource users is that such models do a reasonably good job of capturing the underlying decision process; though resource users and managers may claim that a large variety of factors affect their decisions, the analyst develops a representative model in which the decision process is conditioned by a few key variables. A multitude of other factors may influence the agent's decision, but from the perspective of the analyst these other factors are the unobserved elements of a stochastic process.

In the discussion below I illustrate the essential features of structural estimation of dynamic decision problems by examining a simple model of timber harvesting. I then discuss reduced form estimation of the associated decision rule, and briefly examine the relative merits of structural and reduced form estimation.

STRUCTURAL ESTIMATION OF A DYNAMIC DECISION PROBLEM

To keep the presentation manageable, I focus on the matter of estimating an optimal stopping problem, in which the decision-maker merely chooses whether or not to let an activity continue. Let *i*=0 denote the decision to continue the activity, and let *i*=1 denote the decision to stop. The amount of the payoff in a period depends on the state variables $\mathbf{x}$, which evolve over time according to the probability density function $\mathrm{f}(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$, the decision-specific random shock e^i, and perhaps other variables, denoted by the vector **y**, that are invariant over time**.** The random shocks are contemporaneously observed by the decision-maker but never observed by anyone else; in particular, they are not observed by the analyst. For simplicity we assume these shocks are additive, and identically and independently distributed over time. Letting $\mathrm{R}^{\mathrm{i}}(\mathbf{x}_t, \mathbf{y}) + e_t^i$ denote the decision-specific payoff at time *t*, the relevant decision problem can be stated,

$$V(\mathbf{x}_t, \mathbf{y}) = \max\{\mathrm{R}^0(\mathbf{x}_t, \mathbf{y}) + e_t^0 + qE\{V(\mathbf{x}_{t+1}, \mathbf{y})\}, R^1(\mathbf{x}_t, \mathbf{y}) + e_t^1\}, \tag{1}$$

where q is a discount factor.

The problem in (1) reflects Bellman's principal of optimality; in particular, the expected value of not stopping the activity in period *t* implicitly recognizes that optimal decisions are made in the future. This problem can be solved via the recursive methods of dynamic programming. The solution is an optimal decision rule $i(\mathbf{x}_t, \mathbf{y}, e_t; \Gamma)$, where e_t denotes the difference $e_t^1 - e_t^0$, and Γ is the set of structural parameters associated with the decision problem.

This basic problem can be adapted to the timber harvest decision on even-aged timber stands held in nonindustrial private forests (NIPFs) —those forests held by landowners not directly involved in the manufacture of forest products. The issue of timber harvests on NIPFs has received a considerable amount of attention in the forest

economics literature because nationally these forests comprise about 75% of commercial timberland, and there is concern among forestry professionals that these forests are not well managed.

Formally, let s_t denote the age of the timber stand at time t, and let $\mathrm{u}(s_t)$ denote a money measure of periodic utility from the standing forest. Following the previous literature, the utility received by NIPF owners from the standing forest depends on the age of the timber stand. Also, let p_t and $\mathrm{v}(s_t)$ denote the price and volume of timber, and let l denote the market value of bare forestland —it is the salvage value associated with the decision to harvest the timber stand. It follows that here $\mathbf{x}_t$ is comprised of the state variables p_t and s_t, and

$$\begin{aligned} \mathrm{R}^0(\mathbf{x}_t, \mathbf{y}) &= \mathrm{u}(s_t) \\ \mathrm{R}^1(\mathbf{x}_t, \mathbf{y}) &= \mathrm{u}(s_t) + p_t \mathrm{v}(s_t) + l \end{aligned} .$$

Now suppose there exists observations over time $t=1,\ldots T$, and across individuals (or firms) $j=1,\ldots J$, of decisions i_{jt} and variables $\mathbf{x}_{jt}$ and $\mathbf{y}_j$. The probability of harvest by individual j at time t is defined by

$$\begin{aligned} \Pr(i_{jt}=1) &= \Pr\left(R^1(\mathbf{x}_{jt}, \mathbf{y}_j) - \left[\mathrm{R}^0(\mathbf{x}_{jt}, \mathbf{y}_j) + qE\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\}\right] + e_{jt} > 0\right) \\ &= \Pr\left(e_{jt} > \mathrm{R}^0(\mathbf{x}_{jt}, \mathbf{y}_j) + qE\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\} - R^1(\mathbf{x}_{jt}, \mathbf{y}_j)\right) \\ &= \int_{e>A} g(e \mid s_e) \end{aligned} , \tag{2}$$

where $A = \mathrm{R}^0(\mathbf{x}_{jt}, \mathbf{y}_j) + qE\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\} - R^1(\mathbf{x}_{jt}, \mathbf{y}_j)$, and $g(e \mid s_e)$ is the probability density function of e. Let Γ denote the vector of structural parameters, which includes the discount rate, parameters of the payoff functions $\mathrm{R}^i(\cdot)$, parameters of the distribution function $\mathrm{f}(\cdot)$, and the distribution parameters s_e. Formally, the likelihood function is

$$\begin{aligned} L(\Gamma) &= \prod_{j=1}^{J} \prod_{t=1}^{T} \Pr(i_{jt} \mid \mathbf{x}_{jt}, \mathbf{y}_j; \Gamma) \\ &= \prod_{j=1}^{J} L_j(\Gamma) \end{aligned} . \tag{3}$$

Note that estimation of Γ is a complicated and computationally intensive exercise, much more so than is found with typical static models. The search for the parameter vector that maximizes the likelihood function involves solving the decision problem (1) *each time* new parameter values are evaluated in the search; this is apparent by the presence of $E\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_j)\}$ in the limit of integration in (2). Maximum likelihood estimation thus involves the nesting of an "inner" dynamic programming algorithm within an "outer" hill-climbing algorithm (Rust 1994).

THE REDUCED FORM ALTERNATIVE

Given the difficulty of estimating an appropriate structural model, it would appear reasonable to turn instead to estimation of the dynamic decision rule, $i(\mathbf{x}_t, \mathbf{y}, e_t; \Gamma)$. Probit and logit regression can be rationalized as attempts to do this. One might suppose, for instance, that the decision rule is characterized by a latent variable i^*_{jt}, the value of which is linear in the original variables:

(4) $$i_{jt}^{*} = b_{\mathbf{x}}\mathbf{x}_{jt} + b_{\mathbf{y}}\mathbf{y}_{j} + e_{jt},$$

where $b_{\mathbf{x}}$ and $b_{\mathbf{y}}$ are conformable vectors of parameters. The decision i_{jt} relates to the latent variable i_{jt}^{*} as follows:

(5) $$i_{jt} = \begin{cases} 1 & if \quad i_{jt}^{*} > 0 \\ 0 & otherwise \end{cases},$$

and so letting $g(e \mid s_{e})$ denote the probability distribution function of e, where s_{e} is a set of parameters defining the distribution, the probability of observing termination of the activity by individual *j* at time *t* is given by

(6) $$\begin{aligned} \Pr(i_{jt} = 1) &= \Pr(b_{\mathbf{x}}\mathbf{x}_{jt} + b_{\mathbf{y}}\mathbf{y}_{j} + e_{jt} > 0) \\ &= \Pr(e_{jt} > -b_{\mathbf{x}} \quad \mathbf{x}_{jt} - b_{\mathbf{y}}\mathbf{y}_{j}), \\ &= \int_{e>A} g(e \mid s_{e}) \end{aligned}$$

where $A = -b_{\mathbf{x}} \quad \mathbf{x}_{jt} - b_{\mathbf{y}}\mathbf{y}_{j}$. Now define $\Gamma^{r} = (b_{\mathbf{x}}, b_{\mathbf{y}}, s_{e})$, and let $\Pr(i_{jt} \mid \mathbf{x}_{jt}, \mathbf{y}_{j}, \Gamma^{r})$ denote the probability of observed decision i_{jt}. Then if the random shock is independently and identically distributed over time and across individuals, the associated likelihood function is simply

(7) $$\begin{aligned} L(\Gamma^{r}) &= \prod_{j=1}^{J}\prod_{t=1}^{T} \Pr(i_{jt} \mid \mathbf{x}_{jt}, \mathbf{y}_{j}, \Gamma^{r}) \\ &= \prod_{j=1}^{J} L_{j}(\Gamma^{r}) \end{aligned},$$

REDUCED FORM VS. STRUCTURAL ESTIMATION

Several observations about reduced form estimation are relevant to evaluating its merit. If the analyst is only interested in identifying the decision rule associated with a particular dynamic decision problem, and in particular, wishes to determine what factors affect the dynamic decisions of resource users and managers, then reduced form estimation is certainly an appropriate tool. Moreover, by casting the estimation as the attempt to recover the underlying decision rule of a dynamic decision problem, the analyst establishes grounds for including higher-order terms in the estimation. Indeed, in the case of the simple optimal stopping problem described above, higher-order terms are necessary to avoid misspecification bias. A comparison of the first lines of (2) and (6) reveals that if the structural model is the "true" decision model, then the latent variable in the reduced form model implicitly denotes the expected net gain from stopping the activity, where the expectation is taken over the future value of the activity. It follows that a reduced form model in which the latent variable is linear in the original variables is appropriate *only* if the expected net gain from stopping the activity is itself linear in the original variables. Formally, we require

(8) $$b_{\mathbf{x}}\mathbf{x}_{jt} + b_{\mathbf{y}}\mathbf{y}_{j} \equiv -A \equiv R^{1}(\mathbf{x}_{jt}, \mathbf{y}_{j}) - [\mathrm{R}^{0}(\mathbf{x}_{jt}, \mathbf{y}_{j}) + qE\{V(\mathbf{x}_{j,t+1}, \mathbf{y}_{j})\}],$$

where *A* is the limit of integration in (2) and (6). For our simple model of NIPF owner behavior, (8) can be restated

(9) $$b_{0} + b_{1}s_{jt} + b_{2}p_{t} \equiv -A \equiv p_{t}\,\mathrm{v}(s_{t}) + l - qE\{V(s_{j,t+1}, p_{t+1})\},$$

which states that the expected net gain from harvesting is linear in timber price and stand age. Generally this is not true. Consider, for instance, the case where the price expectations of NIPF owners take the naïve form examined by Brazee and Mendelsohn (1988); in particular, NIPF owners suppose that observed prices are random fluctuations around a fixed mean price. In this case $E\{V(s_{j,t+1}, p_{t+1})\}$ is *not* a function of the current price p_t, and so clearly (9) cannot hold so long as the volume function v(s_t) is anything but a constant. Standard probit estimation omits a relevant dependent variable (total revenue, $p_t \, \mathrm{v}(s_t)$), and includes an irrelevant dependent variable (p_t). Still another problem is the implicit attempt to represent the discounted expected value function $qE\{V(s_{j,t+1})\}$ as a linear function of s_{jt}. In summary, the structural model is implicitly associated with a decision rule $i(\mathbf{x}_t, \mathbf{y}, e_t; \Gamma)$ which, as indicated by the discussion above, one would expect *can* be well-approximated in reduced form estimation.

Still, there would appear to be some merit to stating the structural model representing the underlying decision process, even if the reduced form model is the one eventually estimated, as this could provide some insight to the appropriate estimation procedure, and to the structure of the decision rule (that is, the structure of the reduced form model). Consider, for instance, the case where the analyst draws NIPF harvest data from two regions distinguished by timber prices, such as the coastal plain and piedmont of North Carolina. In reduced form estimation of the harvest decision rule we might expect explanatory (state) variables to include timber price and some measure of the state of the forest stand, such as stand age or volume per acre. In the absence of an explicit linkage between the structural and reduced form models, the analyst is tempted to estimate a *single* reduced form model, combining the data from both regions. But this would be a mistake. The decision problems solved by timber managers in the two regions are different; what the analyst recovers from estimation are coefficients that reflect the "true" reduced form coefficients of neither region. Consider, for instance, simulation results from a timber harvest model similar to that presented by Haight and Holmes. Simulations were run for 150 timber stands from each of two regions distinguished by the stochastic process governing timber prices. In region 1 prices fluctuated randomly around a mean price of $75/mbf, with a standard deviation of $6.25/mbf, and in region 2 prices fluctuated randomly around a mean price of $150/mbf, with a standard deviation of $12.50/mbf. For such a model, the optimal harvest decision rule is characterized by a reservation price strategy in which a stand of age *s* is harvested only if the observed price is greater than the reservation price, $p^*(s)$ (see Brazee and Mendelsohn 1988). It follows that in each region, price has a positive effect on the decision to harvest. Yet if harvest data are combined in a single probit or logit regression, the positive sign on price will be substantially "diluted," because in region 1 (the low price region), harvests are observed at prices for which the optimal decision in the high price region is to not harvest. Indeed, when each region is treated separately in a regression using observations generated by Monte Carlo simulation of 300 timber stands, the sign on price is positive and significant, as expected. On the other hand, when the regions are combined in a single regression using observations generated via Monte Carlo simulation of 300 timber stands (150 from each region), the sign on price is *negative* and nonsignificant. In repeated trials the coefficient on price in the single regression is in the neighborhood of -0.0007, and the t-statistic is in the neighborhood of -0.8. The upshot is that reduced form estimation requires that the analyst fully differentiate among the dynamic decision problems solved by different agents. Explicitly modeling the underlying structural model can serve to clarify such estimation issues.

Suppose instead that the analyst is indeed interested in the structure of the decision process. For instance, with reference to our model of timber harvesting on NIPF's, the analyst wishes to know something about the price expectations held by NIPF owners. It is clear that reduced form estimation would not be adequate to address this question; estimation of the decision rule provides no insights to the underlying structure of the decision problem, and so its value as a policy tool is somewhat limited. For instance, in the case at hand the analyst would be unable to determine whether the price expectations of forest owners are rational, adaptive, or something else altogether, and so the reduced form model would provide no insight to the response of forest owners to changes in tax policy. Or consider the issue considered by MacNair et al., the implicit price of the nontimber goods and services produced

on national forests.[3] The simple timber harvesting model presented above can be modified to consider the more complicated timber management decision supposedly made on national forests. For instance, if we once again assume that the only timber management decision is the harvest decision (here we make this assumption only for the sake of exposition), then the utility functions $u(s_t)$ can be modified to include additional relevant state variables, such as the distance to various aesthetic features such as lakes and streams, and a parametric version of $u(\mathbf{x}_{jt}, y_t)$ is then estimable upon structural estimation of the model using stand-level data. To reduce the complexity of the problem, those parameters identifiable from a simple revenue maximization model can be estimated separately using alternative data. Such parameters would include parameters associated with timber growth functions and timber price expectations. The possible sources of alternative data used to estimate this subset of parameters are several. One is the timber management models used by the Forest Service or other forestry professionals. Another is the professional judgments of consulting foresters. A third source is the timber management behavior of timber firms. The main point is that the complexity of the problem can be reduced significantly by recursive estimation of parameters. In any event, the estimated payoff function $u(\mathbf{x}_{jt}, y_t)$ is the value placed by the Forest Service on nontimber goods and services. Such values are simply not forthcoming from reduced form estimation of the decision rule used by the Forest Service in the harvest decision.

SUMMARY

To date, economic analyses of dynamic resource management problems has been largely normative, though the returns to such analyses appear to be small. This paper suggests that estimating even relatively simple models of dynamic decision problems can yield substantial insights to the forest management behavior of NIPF owners, timber firms, and government. The main obstacle to structural estimation of stochastic dynamic models is the computational complexity of such models. The dynamic decision problems resource managers and resource users *should* solve are typically understood to be quite complex, involving a large number of state variables. Direct estimation of such problems generally would require a large amount of high quality data, and access to a supercomputer. Yet it seems plausible that ultimately, normative considerations aside, resource managers are often guided by relatively simple rules-of-thumb which can be represented by fairly simple, albeit perhaps noisy, dynamic decision models. Reduced form estimation, which is best understood to be an attempt to estimate the decision rule emanating from a decision process, can serve as a diagnostic tool for identifying those state variables which are most important in the formulation of a structural model, but itself provides few insights relevant to public policy.

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