EFFICIENT AND SUSTAINABLE FOREST STOCK AND HARVEST WHEN NONTIMBER BENEFITS ARE VALUED

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ABSTRACT.—This paper investigates the efficient steady-state forest stock and harvest when both timber and nontimber benefits are valued. The results indicate that forest stock should be thickened when nontimber benefit is valued in addition to timber benefit. In addition, the effect of the discount rate and the ratio of the marginal nontimber benefit to the marginal timber benefit on the optimal steady-state stock and harvest is of the same magnitude with opposite directions. The theoretical model is applied to the U.S. coniferous forests.

INTRODUCTION

Many studies have been undertaken to determine the optimal forest rotation length under different scenarios since the advent of the Faustmann formula. Some of these were focused on the optimal rotation length with the consideration of only timber value (Chang 1983; McConnell et al. 1983; Newman et al. 1985). Others searched for the optimal rotation length with the inclusion of both timber and nontimber benefits (Calish et al. 1978; Hartman 1976; van Kooten 1995). These studies have provided important guidelines on when to cut trees in the even-aged plantations. However, their applications in uneven-aged, or natural forests, are limited because age is no longer an appropriate variable under such circumstances. Also, in formulating a forest management plan/policy, particularly in the management of a large-scale (such as a regional or national scale) forest resource, it may be more relevant to determine how much timber should be harvested and what level of the forest stock should be maintained than to know when trees should be cut. This will become increasingly important if nontimber benefits which largely depend on forest stock, are valued, and if sustainability needs to be addressed in forest resource management and

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utilization. Moreover, the optimal steady-state harvest level, unlike the optimal rotation age, can directly provide information on the long-run potential timber supply from a forest.

To this end, this article investigates the optimal forest stock and harvest with and without addition of nontimber benefits to the timber benefits. A theoretical optimal control model of forest management will be formulated. The model will then be solved to find the optimal steady-state forest stock and harvest levels. Finally, the western and eastern U.S. coniferous forests will be used as an empirical example to illustrate our theoretical results.

THE MODEL

Forest production is a joint production process, in which inputs are transferred into multiple outputs (timber and nontimber products). The benefits derived from forests may consist of timber and nontimber benefits. Assume that the timber benefit is a function of the amount of the timber harvested (h), denoted by U(h), and that the nontimber benefit depends on the level of the forest stock (x), denoted by V(x). It is also assumed that only two inputs are involved in the forest production. One is a composite input, silvicultural effort (E) including all inputs except land needed to generate/regenerate and manage the forest for both timber and nontimber benefits. The other is forest land (L).

To develop a model which can be applied to different sizes of forest land without losing the generality, this analysis is based on a unit area of forest production, i.e., L=1. It may be rational to assume that forest production/management is to maximize the present value of net timber and nontimber benefits over an infinite time horizon subject to the constraints of the forest stock. Such a model for a unit area forest land can be formulated as follows:

$$(1)\ Max_{h(t),\,E(t)} \int_0^{\infty} \{U[h(t)] + V[x(t)] - wE(t) - r\} e^{-d\,t}\, dt$$

subject to:

$$(2) \qquad \frac{dx}{dt} = g[x(t), E(t)] - h(t)$$

(3) $x(0) = x_0$

(4) $x(t) \geq 0$

Here x(t) is the forest stock level at time t, h(t) is the timber harvest level at time t, E(t) is the silvicultural effort at time t, w is the per unit cost of the silvicultural effort, r is the rent of the forest land, δ is the continuous discount rate, and x_0 is the initial forest stock level. g(·) is the growth function of the forest stock, which is assumed to be continuous, twice differentiable, and strictly quasi-concave. These assumptions on the forest growth function seem reasonable because previous studies have shown that forest growth per unit area is generally a logistic or quadratic function of its stock (Smith 1962; Binkley and Dykstra 1987). For a unique interior optimum to exist, U(h) and V(x) are assumed to be continuous, twice differentiable, increasing, and quasi-concave in h and x, respectively. Some studies indicated that the nontimber benefit function V(x) may not be increasing or concave (Calish et al. 1978; Swallow et al. 1990). The violation of these two assumptions on V(x) may lead to multiple local optima, from which a global optimum can be identified, but will not affect the first-order necessary conditions for an optimum (local or global). Since this study focuses on the optimal solution and the impact of some economic and financial factors on the optimal steady-state solution, the violation of the increasingness and quasi-concavity of V(x) will not change the main results derived here.

Equation (1) is the objective function which maximizes the net timber and nontimber benefits over an infinite time period. Equation (2) is the forest stock constraint, which implies that the change rate of the forest stock is the difference between net natural growth and harvest. Equation (3) is the initial stock constraint. Inequality (4) is the nonnegativity constraint for the forest stock. Equations (1)-(3) and inequality (4) constitute an optimal control problem with one state variable x(t) and two control variables, h(t) and E(t). Its corresponding present-value Hamiltonian expression is:

(5) $\mathrm{H}[x(t),\ h(t),\ E(t);\ l(t)] = \{U[h(t)] + V[x(t)] - wE(t) - r\}e^{-dt} + l(t)\{g[x(t),\ E(t)] - h(t)\}$

Here λ(t) is the costate variable, indicating the present-value shadow price of the forest stock (x).

The current-value Hamiltonian expression can be written as:

(6) $\tilde{\mathrm{H}}[x(t), h(t), E(t); m(t)] = U[h(t)] + V[x(t)] - wE(t) - r + m(t)\{g[x(t), E(t)] - h(t)\}$

$\mu(t) = \lambda(t)e^{\delta t}$ is the current-value shadow price of the forest stock (x). The first-order optimality conditions of this problem are given by:

(7) $$\frac{\partial \tilde{H}}{\partial h} = U_h - m = 0$$

(8) $$\frac{\partial \tilde{H}}{\partial E} = -w + m\, g_E = 0$$

(9) $$\frac{dm}{dt} - dm = -\frac{\partial \tilde{H}}{\partial x} = -V_x - m\, g_x$$

Reorganizing equations (7) and (8), we have:

(10) $$U_h = m$$

(11) $$w = m g_E$$

Equation (10) suggests that at the optimal harvest level, the marginal timber benefit must be equal to the current-value shadow price of the forest stock. Equation (11) implies that the demand for factor E should be determined by equalizing the price of the factor to the current value of its marginal product.

Equations (2), (3), (9), (10), and (11) , along with inequality (4), constitute a simultaneous equation system. Given functions U(h), V(x), and g(·), and the values of w, δ, and x_0, the optimal paths (traparetrics) of the controls, h(t) and E(t), and state x(t) can be solved analytically or numerically. By setting $\frac{dx}{dt} = \frac{dh}{dt} = \frac{dE}{dt} = 0$, we can solve for the optimal equilibrium, the optimal steady-state solution (x*, h*, E*). For the detailed procedures regarding the solution of an optimal control problem, please refer to Clark (1990) and Conrad and Clark (1987).

OPTIMAL STEADY-STATE FOREST STOCK AND HARVEST

From the theoretical model presented previously, the optimal equilibrium levels of the forest stock and harvest can be derived. In addition to maximizing the present value of the net timber and nontimber benefits, the solution from the above model also represents a sustainable level of the forest stock and harvest because the optimal equilibrium

is a steady-state solution. Therefore, the optimal steady-state forest stock and harvest are efficient and sustainable. Here we discuss two scenarios representing the consideration of (i) only the timber value, and (ii) both the timber and nontimber benefits.

Taking the time derivative of equation (10) and substituting it into equation (9) with U_h substituting for μ (equation (10)), we obtain:

$$\frac{dh}{dt} = \frac{U_h}{U_{hh}}(d - g_x) - \frac{V_x}{U_{hh}} \tag{12}$$

At the equilibrium $\frac{dh}{dt} = 0.$ Thus,

$$U_h(d - g_x) - V_x = 0 \tag{13}$$

That is,

$$d = g_x + \frac{V_x}{U_h} \tag{14}$$

The variable g_x can be interpreted as the marginal growth of the forest stock, the change in the forest growth for each additional unit of its stock. Equation (14) suggests an important relationship among the discount rate, the marginal growth of the forest stock, and the ratio of the marginal nontimber benefit to the marginal timber benefit for the optimal steady-state forest stock and harvest. According to the equation, the optimal steady-state forest stock and harvest should be such that the discount rate is equal to the sum of the marginal growth of the forest stock and the ratio of the marginal nontimber benefit to the marginal timber benefit.

Consideration of Only Timber Value

If only timber is valued, $V_x = 0$. Since U(·) is increasing in h, equation (14) becomes:

$$d = g_x \tag{15}$$

Equation (15) implies that in order to maximize the timber benefit from forest harvests in the long run (at the steady state), we should maintain the forest stock at such a level that the marginal growth of the stock equals the

discount rate. For a quasi-concave growth function g(·), there exists the following relationships between the forest stock (x) and the maximum sustainable yield (MSY) stock (x_{MSY}):

$$(16) \qquad x \begin{matrix} < \\ = \\ > \end{matrix} x_{MSY}, \; when \; g_x \begin{matrix} > \\ = \\ < \end{matrix} 0$$

Hence, when the discount rate approaches zero, g_x also approaches zero, suggesting that the optimal steady-state stock and harvest are equal to the MSY stock and MSY, respectively. As long as the discount rate is positive, $g_x > 0$ at the steady state, indicating that with the consideration of only the timber value, the steady-state optimal forest stock and harvest are smaller than the MSY stock and MSY, respectively. If the forest stock (yield) can be considered to be positively related to its age, this is parallel to the result that the optimal rotation length is shorter than the age corresponding to MSY, founded by many studies on the optimal forest rotation (Bentley and Teeguarden 1965; Samuelson 1976; Hyde 1980; Chang 1983; Clark 1990).

Consideration of Both Timber and Nontimber Benefits

Rearranging equation (14), we have:

$$(17) \qquad g_x = d - \frac{V_x}{U_h}$$

At the optimal equilibrium, g_x could be positive, negative, or zero. If $\delta < V_x/U_h$, or $\delta = 0$, then $g_x < 0$, indicating that the optimal forest stock exceeds the MSY stock, and that the corresponding optimal harvest is smaller than MSY for a quasi-concave g(·). If $\delta = V_x/U_h$, then $g_x = 0$, implying that optimal steady-state forest stock and harvest are equal to the MSY stock and MSY, respectively. If $\delta > V_x/U_h$, $g_x > 0$. In this situation, the optimal forest stock is smaller than the MSY stock. The optimal harvest is smaller than the MSY as well.

Since V(x) and U(h) are increasing in x and h respectively, i.e., V_x and U_h are positive, based on equation (17) we can infer:

(18) $$d > g_x$$

The comparison of equation (15) with equation (18) indicates that at a given discount rate, the optimal steady-state forest stock when both timber and nontimber benefits are valued exceeds that when only the timber is valued. This result is true as long as the forest growth function g(·) is quasi-concave in stock (x). It is independent of the function forms of U(h) and V(x), which could be difficult to quantify (particularly V(x)). Previous studies on the determination of the optimal rotation length (Hartman 1976, van Kooten et al. 1995) showed that the rotation period with the consideration of both timber and nontimber benefits would be longer than that with the consideration of only the timber benefit. For a given area of forest land, a larger forest stock means an older stand as long as the forest stock is below its carrying capacity. Therefore, our result, when applied to even-aged forest plantations, is also consistent with these previous findings. Moreover, the optimal steady-state harvest level when both timber and nontimber benefits are valued is higher (lower) than that when only the timber benefit is considered if g_x valued at the optimal equilibrium is greater (less) than zero. Comparing equation (15) with equation (17) reveals that the effect of V_x/U_h and the discount rate on the optimal forest stock and harvest is of the same magnitude, but has opposite directions. In other words, inclusion of nontimber benefit in the determination of the optimal steady-state forest stock and harvest is equivalent to reducing the discount rate by V_x/U_h.

AN EMPIRICAL EXAMPLE: U.S. CONIFEROUS FORESTS

The theoretical model and conclusions presented above are illustrated by using the western and eastern U.S. coniferous forests as an empirical example. In our theoretical model, the forest growth per unit area, g(x, E) is defined as a function of the forest stock (x) and silvicultural effort (E). Unfortunately, such a forest growth function is rarely available. For the simplicity of the illustration, let us assume that E is fixed. Thus, we consider forest growth as a function of its stock only. Binkley and Dykstra (1987) estimated such forest growth functions for major forests in the world. Their estimated forest growth functions for both the western and eastern U.S. coniferous forests are used for demonstration here. The growth models they estimated are:

- For the western U.S. coniferous forest:

(19) $$g(x) = 0.0625x - 0.0002803x^2$$

- For the eastern U.S. coniferous forest:

$$(20) \qquad g(x) = 0.00493x^2 - 0.0001059x^3$$

Equation (19) is a logistic function, and equation (20) is a cubic function. Both of them are continuous, twice differentiable, and strictly quasi-concave.

Assume that the timber benefit U(h) = ρh, and the nontimber benefit V(x) = ρx. Using equation (2) after dropping E(t) and equations (4), (7), and (9), we can solve for the optimal steady-state forest stock (x*) and harvest (h*) as follows:

- For the western U.S. coniferous forest:

$$(21) \qquad x^* = 1783.803\left(\frac{r}{p} - d + 0.0625\right)$$

$$(22) \qquad h^* = 1783.803\left(\frac{r}{p} - d + 0.0625\right)\left[0.0625 - 0.5\left(\frac{r}{p} - d + 0.0625\right)\right]$$

- For the eastern U.S. coniferous forest:

$$(23) \qquad x^* = 1573.812\left\{0.009862 + \left[0.000097259 - 0.00127176\left(d - \frac{r}{p}\right)\right]^{0.5}\right\}$$

$$(24) \qquad h^* = x^{*2}\left(0.004931 - 0.0001059x^*\right)$$

The optimal steady-state forest stock and harvest for the western and eastern U.S. coniferous forests under various levels of the ρ/p ratio and the discount rate (δ) according to equations (21), (22), (23), and (24) are shown in tables 1 and 2, respectively. These two tables suggest how much to cut and to remain in the long run (at the steady state) for one acre of the western and eastern conifers at a given discount rate and a specific ratio of the marginal nontimber benefit to the marginal timber benefit.

Based on the forest growth functions (equations (19) and (20)), the maximum forest stock (carrying capacity) is 222.98 m^3/ac (cubic meters per acre) for the western coniferous forests and 46.56 m^3/ac for the eastern coniferous

forests. The maximum sustainable yield (MSY) and the MSY stock are respectively 3.48 m^3/ac/yr (cubic meters per acre per year) and 111.49 m^3/ac for the western conifers, and 1.58 m^3/ac/yr and 31.04 m^3/ac for the eastern conifers.

Tables 1 and 2 show that for both the western and eastern conifers, if nontimber benefit is not valued, the optimal steady-state stock levels are smaller than their MSY stocks, and the optimal steady-state harvest levels are also smaller than MSY. Also, without the consideration of nontimber benefit, the optimal steady-state forest stock and harvest decrease as the discount rate increases. As the discount rate approaches zero, the optimal steady-state stock and harvest approach the MSY stock and MSY, respectively. Inclusion of nontimber benefit increases the optimal steady-state stock at a specific discount rate. However, even with the consideration of nontimber benefit, the optimal steady-state stock and harvest can be smaller than, equal to, or larger than the MSY stock and MSY, respectively, depending upon the sign of $\rho/p - \delta$. As long as $\rho/p = \delta$, the optimal steady-state stock should be equal to the MSY stock and the optimal steady-state harvest should equal the MSY. When $\rho/p > \delta$, the optimal steady-state harvest should be smaller than MSY, but the optimal steady-state stock should be maintained at a level larger than the MSY stock. If ρ/p is high enough relative to the discount rate, no timber should be harvested, and the forest stock will reach its carrying capacity. For example, at $\delta = 4$ percent no timber should be harvested from the western U.S. coniferous forests if $\rho/p \geq 0.12$ (table 1), and from the eastern U.S. coniferous forests if $\rho/p \geq 0.28$ (table 2). When $\rho/p < \delta$, the optimal steady-state stock and harvest should be smaller than MSY and the MSY stock, respectively. On the other hand, if δ is high enough relative to ρ/p, the forests should be cleared. According to tables 1 and 2, if the nontimber benefit is not valued, both the western and eastern conifers would be cleared if the real continuous discount rate is 8 percent or higher. These two tables also illustrate that an increase in ρ/p is equivalent to deflating the discount rate by the same amount in terms of the impact on the optimal steady-state stock and harvest. For any combinations of δ and ρ/p, as long as $\rho/p - \delta$ is equal to the same value, their corresponding steady-state forest stock and harvest are identical.

As described previously, the optimal steady-state harvest is actually the long-run sustainable timber supply. If nontimber benefit is not valued, the western coniferous forest will supply 2.06 m^3/ac/yr of timber, and the eastern

coniferous forest 1.48 m^3/ac/yr at a 4 percent real discount rate in the long run. The long-run timber supply levels (equal to h*) for the western and eastern U.S. coniferous forests at various ρ/p ratios and discount rates can be found in tables 1 and 2.

The current average annual timber removal is about 0.6 m^3/ac from the western coniferous forest and 1.4 m^3/ac from the eastern coniferous forest (Waddell et al., 1989). If the current state of these forests can be thought to be around their optimal steady states, the ρ/p ratio corresponding to the current timber removal level for either the western or eastern U.S. coniferous forests is approximately equal to 0.1 (tables 1 and 2). This implies that the nontimber benefit we valued is about 10 percent of the timber prices for both the western and eastern U.S. coniferous forests.

SUMMARY AND CONCLUSIONS

Information on how much to cut (harvest) and to remain (stock), like that on when to cut (rotation age), is important in forest management. Using forest stock and harvest levels as decision variables is applicable to the management of both plantations and natural forests. This paper describes an approach to determining the optimal forest stock and harvest and demonstrates the applicability of this approach in empirical forest management using the example of the U.S. coniferous forests. Our results indicate that the optimal steady-state forest stock increases when nontimber benefit is added to timber benefit. The optimal steady-state stock does not exceed the MSY stock when only timber is valued, but it can be smaller than, equal to, or larger than the MSY stock when both timber and nontimber benefits are valued. The impact of the inclusion of nontimber benefit on the optimal stock and harvest is the same as that of a decrease in the discount rate by the ratio of the marginal nontimber benefit to the marginal timber benefit (V_x/U_h). Too high a discount rate (relative to V_x/U_h) could lead to deforestation. A high value of V_x/U_h can help conserve forest resources.

The results from our empirical examples have several potential applications and implications. First, the optimal timber harvest levels in tables 1 and 2 can be used to project the long-run timber supply from the western and

eastern U.S. coniferous forests. Second, the current timber removal level is close to its MSY in the eastern coniferous forest, but much below its MSY in the western coniferous forest. Therefore, the maximum sustainable yield is at least not the only criterion we currently use to manage the western conifers. Third, the ratio of the marginal nontimber benefit to the marginal timber benefit corresponding to the current timber removal level in both the eastern and western coniferous forests is around 0.1 at a 4 percent discount rate. Our current behavior in managing these forests implicitly reveals that the nontimber benefit of these conifers in general is valued at about 10 percent of their timber prices (the marginal timber benefit).

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Table 1. The optimal steady state forest stock (x*) and harvest (h*) per unit acre under various values of δ and ρ/p for the western U.S. coniferous forests.

	δ = 0%		δ = 2%		δ = 4%		δ = 6%		δ = 8%		δ = 10%	
ρ/p	x*	h*	x*	h*	x*	h*	x*	h*	x*	h*	x*	h*
0.000	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273	0.000	0.000	0.000	0.000
0.010	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254	0.000	0.000	0.000	0.000
0.020	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273	0.000	0.000
0.030	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254	0.000	0.000
0.040	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057	4.460	0.273
0.050	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681	22.298	1.254
0.060	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127	40.136	2.057
0.070	222.975	0.000	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395	57.974	2.681
0.080	222.975	0.000	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484	75.812	3.127
0.090	222.975	0.000	222.975	0.000	200.678	1.254	165.002	2.681	129.326	3.395	93.650	3.395
0.100	222.975	0.000	222.975	0.000	218.516	0.273	182.840	2.057	147.164	3.127	111.488	3.484
0.150	222.975	0.000	222.975	0.000	222.975	0.000	222.975	0.000	222.975	0.000	200.678	1.254
0.200	222.975	0.000	222.975	0.000	222.975	0.000	222.975	0.000	222.975	0.000	222.975	0.000

Note: The growth model of the western U.S. coniferous forests is from Binkley and Dykstra (1987). The units of x* and h* are m^3/acre and m^3/acre/year, respectively.

Table 2. The optimal steady state forest stock (x*) and harvest (h*) per unit acre under various values of δ and ρ/p for the eastern U.S. coniferous forests.

	δ= 0%		δ = 2%		δ = 4%		δ = 6%		δ = 8%		δ = 10%	
ρ /p	x*	h*	x*	h*	x*	h*	x*	h*	x*	h*	x*	h*
0.00	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304	0.000	0.000	0.000	0.000
0.01	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410	20.037	1.128	0.000	0.000
0.02	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304	0.000	0.000
0.03	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410	20.037	1.128
0.04	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482	22.725	1.304
0.05	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530	24.653	1.410
0.06	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561	26.240	1.482
0.07	37.001	1.386	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579	27.620	1.530
0.08	37.722	1.332	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584	28.859	1.561
0.09	38.421	1.273	37.001	1.386	35.481	1.477	33.835	1.543	32.025	1.579	29.992	1.579
0.10	39.098	1.208	37.722	1.332	36.255	1.435	34.676	1.514	32.954	1.565	31.042	1.584
0.15	42.230	0.818	41.024	0.987	39.757	1.139	38.421	1.273	37.001	1.386	35.481	1.477
0.20	45.032	0.329	43.945	0.535	42.814	0.728	41.634	0.905	40.399	1.065	39.098	1.208
0.25	46.563	0.000	46.563	0.000	45.561	0.220	44.493	0.434	43.385	0.633	42.230	0.818
0.30	46.563	0.000	46.563	0.000	46.563	0.000	46.563	0.000	46.081	0.108	45.032	0.329
0.35	46.563	0.000	46.563	0.000	46.563	0.000	46.563	0.000	46.563	0.000	46.563	0.000

Note: The growth model of the eastern U.S. coniferous forests is from Binkley and Dykstra (1987). The units of x* and h* are m^3/acre and m^3/acre/year, respectively.