EXPLORING ENHANCEMENTS TO DYNAMIC PROGRAMMING

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ABSTRACT.—Dynamic programming (DP) has long been used to solve the optimal rotation length and density problem in timber management. Recent modifications or enhancements to standard forward recursive DP include using "look-aheads," where future growth potential as well as current state values are compared explicitly to determine the optimal network path. The use of such look-aheads to completely solve one stage of the DP problem at a time has also been considered and is known as the PATH algorithm. DP has also been enhanced through the use of heuristics where state coefficients and entering state variable values are used to modify the objective function value in an attempt to favor stand attributes that are known to produce high-value rotations. This research will compare standard forward recursive DP to "look-ahead" assisted DP, heuristics-aided DP, and a PATH algorithm. The platform for comparison will be a forward-recursive DP model utilizing an individual-tree growth projection system. Criteria for comparison include required size of network formulation, objective function values, and required solution time.

INTRODUCTION

Dynamic programming (DP) has long been used to solve the optimal rotation length and density problem in forest management (Amidon and Akin 1968; Schreuder 1971; Haight et al. 1985). However, DP formulations become intractable in size quickly due to the use of multiple, continuous state variables (Brodie and Kao 1979; Arthaud 1986). Several methods of "enhancing" DP formulations have been made to DP and will be discussed briefly below.

The first enhancement to DP is the use of "neighborhood storage" (Brodie and Kao, 1979). This involves converting continuous state variables into state "neighborhoods" in which all entering states are considered the same in terms of future growth potential. Then only the state with the highest accumulated value is maintained. The formulation for this method is shown below:

(1)
$$f_N(Y_N) = \sum_{n=0}^{N} r_n(T_n)$$

(2)
$$Y_n + G_{n+1}(Y_n) - T_n = Y_{n+1} (n = 0,1,2,\ldots,N-1)$$

(3)
$$X_N - T_N = 0$$

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(4)

$$f_n(Y_n) = \max_{Y_{n-1},T_n} [r_n(X_n,T_n) + f_{n-1}(Y_{n-1})]$$

Equation 4 is the standard forward recursive DP equation that is used. Y_{n-1} represents a state vector that can be modeled by discrete states or continuous states or state neighborhoods as is the case in this study.

Paredes and Brodie (1987) and Yoshimoto et al. (1990) suggested that a growth look-ahead might be used to explicitly determine the value of all future growth from a particular state. Using a growth look-ahead to augment the DP formulation would change the recursive equation (4) to:

(5)

$$f_n(Y_n) = \max_{Y_{n-1},T_n} [r_n(X_n,T_n) + r_L G_L(Y_n) + f_{n-1}(Y_{n-1})]$$

In equation 5, each state (Y_n) is projected forward L years by the function $G_L(Y_n)$ and a return function r_L converts this growth to volume and value. Thus, the accumulated value plus the explicit projected value of the state entering a neighborhood is compared to the highest current value occupying the neighborhood with the highest value being retained.

Paredes and Brodie (1987) suggested that if the growth function is monotonically increasing, then an entire stage can be solved at once using the growth look-ahead and changing the recursive relationship to:

(6)

$$f_n(Y_n) = \max_{T_n} [r_n(X_n,T_n) + r_L G_L(Y_n) + f_{n-1}(Y_{n-1})]$$

In this case, only the highest value state in each stage is saved for projection in future stages, under the assumption that the growth projection provides enough information about the future value of all states in a stage that the optimal can be chosen.

The final enhancement to DP is the use of heuristics (Arthaud and Pelkki 1996). The process is very similar to the growth look-ahead, but instead of a growth function, a simple heuristic equation is used to augment the accumulated value and favor certain intra-state conditions. The recursive relationship and heuristic equation used in this study are:

(7)

$$f_n(Y_n) = \max_{Y_{n-1},T_n} [r_n(X_n,T_n) + H(Y_n) + f_{n-1}(Ysubn-1)]$$

The coefficients (b_1 and b_2) can weigh intrastate conditions with higher average diameter (DBH) and number of trees (NT) as the user desires.

(8)
$$H(Y_n) = b_1(\overline{DBH}) + b_2(NT)$$

This study examines each of these techniques on a single problem formulation and evaluates them in terms of objective function value found and memory and processing time requirements.

METHODS AND DATA

The DP and enhanced DP simulations were completed on a hypothetical yellow poplar stand. The objective function of all simulations was to maximize soil expectation value. The stand input data was derived from regional diameter distributions for a 20-year old site of yellow poplar with an index of 110 (Knoebel et al. 1986; McGee and Della-Bianca 1967). This data was selected because the growth projection system used is based on the GROW subroutine (Brand 1981) with northeast regional parameters (Yaussy and Gale 1992). Tree quality projections were made according to the methods in Yaussy (1993) with regional tree quality averages coming from the Eastwide Forest Inventory Database (Hansen et al. 1992) As stated, a forward recursive DP algorithm was employed with a stage interval of 2 years.

Five different thinning types (above, below, above and below, improvement, and high-grade) were included with removal intensities running from 10-50 percent in 5 percent increments in all simulations. An $80 per acre fixed entry cost was charged on all harvest entries and thinning revenue was penalized 15 percent for variable thinning costs. Stumpage prices for roundwood and sawtimber by grade were derived from regional price reports. A 4 percent real rate of return was used as the cost of capital in all simulations.

All combinations of two-state parameter DP simulations were tested using the state variables of basal area (BA), number of trees (NT), cubic foot volume (VOL), and average diameter (DBH). These two-state DP models were tested with various state neighborhood sizes to observe improvements in the objective function value with decreases in state neighborhood size.

Once the DP simulations were completed, a known suboptimal state neighborhood combination was tested with the DP+look-ahead formulation. This was done to see if any of the enhancements to DP could make up for state neighborhoods that definitely violated the principle of optimality. Also, it was desirable to start with state neighborhoods that were large and resulted in very fast DP solution times since many runs would be tested. First, various lengths of growth look-ahead were tested to see which, if any, produced the best results. Then the best growth look-ahead was used with a variety of DP state neighborhood combinations to see how much the objective function value could be improved.

The next step was to apply the PATH algorithm. As mentioned above, PATH retains only one state per stage. The state retained is the one with the highest accumulated and future growth value. Therefore, state neighborhood size is immaterial, and several growth look-ahead periods were evaluated to see how PATH compared to other methods.

The final DP enhancement tested was DP+heuristic. As shown above, a simple heuristic was tested by using a binary search to find heuristic parameters (b_1 and b_2) that would improve the DP formulation. The DP+heuristic was tested in the same manner as the DP+look-ahead.

RESULTS AND DISCUSSION

Out of the six combinations of state variables, only the results from BA-VOL and DBH-NT are shown due to space

limitations. Tables 1 and 2 depict results for BA-VOL and DBH-NT, respectively. The best objective function value found in any state neighborhood combination was $326.2, although as indicated in table 2, not every combination was able to find this solution. In general, as state neighborhood size decreases, the objective function value increases. The state neighborhood combination of DBH = 0.5" and NT = 5 trees per acre was selected for testing the DP+look-ahead and DP+heuristic enhancements.

Table 1.—Results of DP only simulations with various basal area (BA) and cubic foot volume per acre (VOL) state neighborhoods

BA state neighborhood	Cubic foot volume per acre state neighborhood				
	6	10	20	40	80
inches					
0.1	insuff. memory	insuff. memory	**324.7**[a] *452301*[b]	**324.4** *226926*	**320.8** *115264*
0.2	**326.2** *752335*	**324.7** *451401*	**324.4** *226151*	**324.4** *113525*	**311.0** *57664*
0.5	**322.9** *301435*	**322.9** *180861*	**322.0** *90611*	**311.0** *45486*	**311.0** *23104*
1.0	**324.4** *151135*	**322.0** *90861*	**322.0** *45431*	**322.0** *22806*	**280.5** *11584*
2.0	**322.0** *75985*	**322.0** *45591*	**322.0** *22841*	**322.0** *11466*	**283.1** *5824*

[a]Figures in bold are objective function values (Soil Expectation Value (SEV) in $/ac).
[b]Figures in italics are the numbers of state neighborhoods required per stage.

Table 2.—Results of DP only simulations with various average diameter (DBH) and number of trees per acre state neighborhoods

DBH state neighborhood	Number of trees per acre state neighborhood				
	1	2	5	10	20
inches					
0.1	**313.5**[a] *180851*[b]	**313.5** *90651*	**310.3** *36531*	**294.2** *18491*	**294.2** *9471*
0.2	**320.5** *90626*	**320.5** *45426*	**294.2** *18306*	**278.2** *9266*	**278.2** *4746*
0.5	**310.3** *36491*	**310.3** *18291*	**281.4** *7371*	**278.9** *3731*	**270.6** *1911*
1.0	**287.0** *18446*	**292.6** *9246*	**278.2** *3726*	**278.2** *1886*	**249.0** *966*
2.0	**278.2** *9746*	**278.2** *4824*	**278.2** *1944*	**278.2** *984*	**238.4** *504*

[a]Figures in bold are objective function values (Soil Expectation Value (SEV) in $/ac).
[b]Figures in italics are number of state neighborhoods required per stage.

Table 3 shows various growth look-ahead lengths applied to the basic DP formulation. From this it is obvious that the look-ahead is of tremendous value and explicit calculation of stand value improves upon DP solutions with much smaller state neighborhoods. However, very long projections of 20, while improving upon a suboptimal DP formulation, do not yield an objective function as high as DP alone.

Table 3.—Testing various look-ahead periods on DP state neighborhoods where DBH=0.5 inches and NT = 5/ac.

Look-ahead period	Objective Function Value	Time to complete	Number of required Stages
	$/acre	*seconds*	
NONE (DP only)	281.4	3450	26
0	327.2	6373	40
1	327.1	7377	40
2	329.3	8069	36
5	329.3	7931	36
10	329.0	9362	35
20	324.6	14047	40

The results of taking the optimal look-ahead period of 5 years and applying it to various state neighborhood sizes is shown in Table 4. The growth projection requires two to three times more processing time but no additional memory, and all DP+look-ahead combinations are better than DP alone. Most importantly, the DP+look-ahead finds a higher valued objective function with far less memory requirements (but more processing time) than DP alone.

Table 4.—Testing a 5-year look-ahead on various DBH-NT state neighborhoods and comparing to straight DP.

State Neighborhoods			Results w/o look-ahead			Results w/Look-ahead		
DBH	NT	# States	SEV	Time	Stages	SEV	Time	Stages
inches			*$/acre*	*seconds*		*$/acre*	*seconds*	
0.2	1	90626	320.3	38597	26	329.6	107758	40
0.2	2	45426	320.3	19726	26	329.6	51564	40
0.2	5	18306	294.2	8633	27	329.3	20443	36
0.5	5	7371	281.4	3450	26	329.3	8069	36
0.5	10	3731	279.9	2286	36	327.2	4067	36
1.0	10	1886	278.2	830	24	322.9	1694	26
1.0	20	966	249.0	417	30	322.9	905	26
2.0	20	504	238.4	230	33	322.9	518	26

The PATH algorithm was tested using growth look-aheads of 0 to 20 years. The results are very disappointing (Table 5). Only the five year look-ahead performed better than DP alone. While PATH is very fast, unless the proper look-ahead length is used, it produces inferior results to straight DP.

Table 5.—Testing various growth look-aheads using the PATH algorithm

Length of look-ahead	Objective Function Value	Time to solve	Stages
years	*$/acre*	*seconds*	
0	225.1	464	24
1	225.1	492	24
2	225.1	467	24
5	320.8	537	26
10	261.2	664	34
20	245.8	572	28

Tables 6 and 7 show the results of adding a simple heuristic to the DP formulation. While the heuristic does not require much additional processing time, the only improvements to DP occur when state neighborhoods are very large and grossly violate the principle of optimality.

Table 6.—Testing various heuristic parameters on DP state neighborhoods DBH=0.5" and NT = 5/acre

b_1	b_2	Objective Function. Value	Time	# of stages
		$/acre	*seconds*	

---	---	281.4	3450	26
4	0	281.4	3592	26
8	0	281.4	3551	26
16	0	288.0	4773	35
32	0	281.4	3914	26
0	-1.0	281.4	3570	26
0	-2.0	289.7	3793	27
0	-4.0	289.7	3958	27
0	-8.0	289.7	4011	27
16	-2.0	289.7	3665	27
0	4	294.2	3787	27
16	4	294.2	3733	27
20	0	278.2	3447	26

Table 7.—Testing the best heuristic found (b_1 = 0, b_2 = 4) to various DBH-NT state neighborhoods and comparing to straight DP

State Neighborhoods			Results without look-ahead			Results with Look-ahead		
DBH	NT	# States	SEV	Time	Stages	SEV	Time	Stages
inches			*$/acre*	*seconds*		*$/acre*	*seconds*	
0.2	1	90626	320.3	38597	26	320.3	37887	26
0.2	2	45426	320.3	19726	26	320.3	21283	26
0.2	5	18306	294.2	8633	27	294.2	8666	27
0.5	5	7371	281.4	3450	26	294.2	3787	27
0.5	10	3731	279.9	2286	36	280.0	1743	25
1.0	10	1886	278.2	830	24	278.2	916	24
1.0	20	966	249.0	417	30	270.3	511	23
2.0	20	504	238.4	230	33	270.3	302	23

CONCLUSION

It is apparent that adding a growth look-ahead to the DP formulation can improve the value of the objective function without additional memory requirements but at a cost of two to three times more processing time. PATH is very fast and can provide “good” results, but obviously, the growth function is not monotonic. Thus, PATH cannot find solutions as good as DP or DP+look-ahead. The DP+heuristic is marginally effective on suboptimal DP neighborhoods, but the testing here was very rudimentary. It may be possible to devise better heuristics that will perform better than the simple ones here.

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