A COMPARISON OF TWO GROWTH MODELS DEVELOPED FOR REGIONAL TIMBER INVENTORY PROJECTION

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ABSTRACT.—This paper compares the inventory growth components of two timber supply models: STRIPS and RXWRITE. STRIPS uses a plot-level growth model, and RXWRITE uses a tree-level growth model. Both models were developed with USDA Forest Inventory and Analysis (FIA) data for Louisiana. They were calibrated by projecting undisturbed plots from the 1984 survey to 1991. Projected plot data were compared to the observed 1991 plot data. The models' plot volume root mean squared prediction errors are compared for six forest types and four species groups. In almost all cases, the STRIPS model performed better than the RXWRITE model.

INTRODUCTION

Projecting regional timber inventories requires the prediction of three fundamental types of change: growth, removals and area change. This paper focuses on the components that predict inventory growth for two timber supply models: STRIPS and RXWRITE. The starting point, of course, for any inventory projection system is the current forest inventory. The primary source of regional timber inventory data in the US is the USDA Forest Service's Forest Inventory and Analysis (FIA) data. In addition to providing starting inventory information, FIA data were used to develop the growth models discussed here. The FIA survey was not designed with this purpose in mind, and, consequently, there are many disadvantages to using FIA data for fitting growth models. In spite of this, FIA data have one key redeeming quality which makes them especially useful for this purpose: only the FIA data are representative of all conditions found in a region. Generally, growth models are developed using data from research plots, which are typically on public or industry land. Because of this selection bias, most growth models tend to over-predict growth on the average forest property — especially non-industrial private properties. Using FIA data to develop growth models eliminates this source of bias. In addition, while FIA data sets tend to be very noisy, they also are usually quite large, which helps to make up somewhat for what they lack in quality.

The objective of this study was to compare how well the two growth models predict the future characteristics of undisturbed FIA plots based on the characteristics of those plots today. STRIPS uses a plot-level growth model, and RXWRITE uses a tree-level growth model. The models were fitted with data from the 1974, 1984 and 1991 FIA surveys of Louisiana. They were then calibrated by projecting undisturbed plots from the 1984 survey to 1991 and comparing the results to the actual plot data observed in 1991. Calibration parameters are iteratively re-estimated until the models provide unbiased predictions of a selected set of plot and/or tree characteristics. After calibration, the models were compared by calculating the root mean square errors of their predictions of plot volume by forest type and species group.

THE TIMBER SUPPLY MODELS

STRIPS

The growth models compared here were developed as part of two very different timber supply models. The first, STRIPS (Tucker and McDill 1996), projects the volume of growing stock[2] in four species groups on individual FIA plots. STRIPS uses plot-level growth equations to predict annual growth, which it adds each year to the existing volume

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[2] A two-product version of STRIPS that projects both growing stock and sawtimber has been developed, but only the one-product model is discussed here.

on the plot in each of four species groups. STRIPS uses 22 growth equations — for each of 6 forest types, 3 to 4 models were developed to predict the growth in each species group found on that forest type. Removals are modeled as clearcuts or partial harvests of plots using historical harvest proportions estimated from the most recent forest survey cycle. These historical harvest proportions are adjusted up or down by a fixed amount so that the resulting harvest matches an exogenously specified total removals target for each year. Changes in the total area of forest land are simulated by adjusting the area expansion factors associated with each plot. Area shifts among forest types are accomplished with Markovian transition matrices estimated from the most recent forest survey cycle. Four transition matrices are constructed for 1) clearcut plots, 2) partially harvested plots, 3) undisturbed plots with less than 500 cu. ft. per acre, and 4) undisturbed plots with more than 500 cu. ft. per acre.

RXWRITE/GISTRAN/DTRAN

The second timber supply model, the RXWRITE/GISTRAN/DTRAN system (Rose et al. 1992, McDill 1992, 1993), projects the state of the forest by choosing management alternatives for each FIA plot to minimize the cost of meeting projected mill requirements. Growth and removals are determined by the management alternative that is selected for each plot by the optimization algorithm. Similarly, forest type change is determined through the selection of regeneration options for each plot. Unlike STRIPS, in the RXWRITE/GISTRAN/DTRAN system it is only possible for the forest type of a plot to change when the plot is clearcut. RXWRITE is a prescription simulator used to develop management alternatives for the system. RXWRITE was originally developed for the Lake States and was written to use STEMS (Belcher 1981, Brand 1981) to model plot growth. Because STEMS is a tree-level growth model, RXWRITE is able to model a variety of thinning and product merchandizing options. A tree-level growth model was developed for Louisiana to retain these advantages in the Southern version of RXWRITE.

THE GROWTH MODELS

STRIPS

As discussed earlier, STRIPS uses three to four growth models for each forest type — one for each species group found on that forest type. The four species groups are: pine, cypress, soft hardwoods, and hard hardwoods. The six forest types are: longleaf/slash pine, loblolly/shortleaf pine, oak-pine, oak-hickory, oak-gum-cypress, and elm-ash-cottonwood. No cypress volume occurs on longleaf/slash pine types, and no pine occurs on elm-ash-cottonwood. The annual growth in each species group is modeled as a function of the total volume on the plot, the volume on the plot of the particular species group being modeled, the site class, and the plot ownership (public, industry, or non-industrial private) and origin (planted or natural — only used on pine and oak-pine types). The general model form is:

$$\Delta Vol_i = B_0 + B_1 Vol_t + B_2 Vol_t^2 + B_3 Vol_i + B_4 Vol_i^2 + B_5 S + B_6 Org + B_7 Pub + B_8 Ind$$
$$+ B_9 Org\, Vol_t + B_{10} Pub\, Vol_t + B_{11} Ind\, Vol_t + B_{12} Org\, Vol_i + B_{13} Pub\, Vol_i + B_{14} Ind\, Vol_i$$

where Vol_t = the total volume on the plot;
Vol_i = the volume on the plot in species group i;
S = the plot site class[3];
Org = a dummy variable set to 1 if the plot is planted (only used on pine and oak-pine);
Pub = a dummy variable set to 1 if the plot is on publicly-owned land; and
Ind = a dummy variable set to 1 if the plot is on land owned by forest industry.

Equation (1) was fitted to data from undisturbed FIA plots from the 1974-1984 and 1984-1991 survey cycles of Louisiana. A backward stepwise procedure was used to find the best model with all parameter estimates significant at

[3]The site class variable in the FIA data is coded with 1 for the best sites and 7 for the worst sites. The variable used here is eight minus the FIA site class variable so that higher values will indicate better sites.

least at the 0.10 level of significance. It is notable that site class was not a significant variable in the majority of cases, and most of the time when site was significant, the coefficient was negative. When the site coefficient was negative, site class was removed from the equation. Space limitations prevent listing the coefficients and their standard errors for all 22 equations. Table 1 shows the number of observations and the r-squared for each model. The r-squares were low, ranging from as low as 0.033 to 0.535.

Table 1.—STRIPS growth model r-squares

Forest Type	Number of Observations	Pine	Cypress	Soft Hardwood	Hard Hardwood
Longleaf/Slash Pine	119	0.149	n.a.	0.377	0.535
Loblolly/Shortleaf Pine	1,070	0.197	0.300	0.170	0.344
Oak-Pine	573	0.071	0.033	0.089	0.168
Oak-Hickory	614	0.110	0.163	0.212	0.167
Oak-Gum-Cypress	2,368	0.270	0.202	0.107	0.201
Elm-Ash-Cottonwood	205	n.a.	0.464	0.128	0.488

RXWRITE.

The state of a tree in the RXWRITE database is described by its species, height, diameter (DBH), and crown ratio. Also associated with each tree record is an expansion factor, which indicates the number of trees per acre represented by the tree. Projection equations are needed for each of these attributes (except species). In RXWRITE, height and DBH are projected with growth equations, crown ratio is projected with a change equation, and the tree expansion factor is projected with a mortality equation. In addition, height and crown ratio were not available for trees with a DBH less than 5 inches. Therefore, height and crown ratio models were developed to predict initial values for these attributes. Due to space limitations, those models are not discussed further here.

In order to be useful for projections, the prediction equations had to use only tree and plot-level variables that would be projected by the system. The available set of predictor variables was:

- tree variables: species (Spp), height (Ht), diameter (DBH), and crown ratio (CR);
- plot variables: site class (S) and basal area (BA);
- tree/plot variable: tree diameter over the plot quadratic mean diameter (DOQ).

Height and crown ratio were not measured in 1974, so only data from the 1984-1991 survey cycle were used to fit the RXWRITE model. The general form of the four tree development equations are:

$$\Delta Ht = f\,(Ht, DBH, CR, S, BA, DOQ\,)$$

$$\Delta DBH = f\,(\,Ht, DBH, CR, S, BA, DOQ\,)$$

$$\Delta CR = f\,(\,Ht, DBH, CR, S, BA, DOQ\,)$$

$$Prob\,(Mortality) = f\,(\,Ht, DBH, CR, S, BA, DOQ\,)$$

These equation were estimated for 18 species groups. The specific model forms used in these equations varied from one species group to another and cannot be described here due to space limits. Both linear and nonlinear models were used. Table 2 lists the species groups and the number of observations and r-squares of each model. In general, the r-squares are low, which should not be surprising, given the use of differenced data and the noisy nature of FIA data. Surprisingly,

the r-squares tended to be highest for the crown ratio change models. The mortality models are logit models, and the r-square has a different interpretation for those models. Roughly speaking, it represents the increase in the proportion of correct predictions with the fitted model as compared with a naive model with no explanatory variables. As with the STRIPS growth model, site class was seldom a good predictor of tree growth.

Table 2.—RXWRITE growth model r-squares

Species Group	No. of Obs.	Height	Diameter	Crown Ratio	No. of Obs.	Mortality
Slash/Longleaf Pine	714	0.160	0.298	0.457	167	0.160
Loblolly Pine	3,353	0.304	0.421	0.526	1,993	0.041
Shortleaf/Spruce Pine	535	0.040	0.308	0.310	325	0.104
Baldcypress	707	0.043	0.080	0.460	1,302	0.043
Red cedar	6	0.855	0.873	0.999	4	n.a.
Tupelo	381	0.078	0.146	0.503	1,201	0.068
Gums/Yellow-poplar	1,143	0.070	0.123	0.385	1,143	0.034
Willow/Box elder/Sweetbay [a]	331	0.111	0.403	0.381		
Willow					434	0.134
Box elder and River Birch					100	0.423
Sweetbay, etc.					169	0.280
Red Maple/Sugarberry/Elm	702	0.062	0.258	0.445	1,289	0.091
Oak Group 1	679	0.070	0.094	0.309	897	0.029
Oak Group 2	1,367	0.083	0.158	0.344	1,472	0.029
Water Hickory/Dogwood/Persimmon	225	0.207	0.542	0.488	453	0.118
Hickory/Pecan	173	0.313	0.135	0.310	219	0.179
Beech	142	0.237	0.528	0.548	183	0.106
Ash/Walnut	269	0.209	0.239	0.407	588	0.127
General Noncommercial	160	0.174	0.304	0.257	332	0.096

[a] Separate mortality models were fit for each sub-group within this species group.

CALIBRATION

It is an eye-opening exercise to project the state of permanent plots from one measurement to a later remeasurement using a growth model that has been fit statistically with data from those very plots. Without calibration, the models will typically produce biased estimates of the plot characteristics of the remeasured plots. This was the case with the models discussed here. As an example, the uncalibrated STRIPS model under-projected the volume on loblolly/ shortleaf pine plots. Calibration parameters can be used to either increase or decrease the change in specific plot or tree attributes over the projection so that the model produces unbiased projections of those attributes. For example, in STRIPS each growth estimate for pine on loblolly/shortleaf pine plots can be multiplied by some constant, say 1.1, to increase the projected volume of pine on loblolly/shortleaf pine plots at the end of the projection. The appropriate values for these calibration parameters were calculated by repeatedly projecting plot data for undisturbed plots from 1984 to 1991 and comparing the characteristics of the projected plot and tree data with the observed characteristics of those plots in 1991. The calibration parameters are re-estimated iteratively until the projected values of these selected plot characteristics are unbiased. Newton's method, with some modifications, was used to re-estimate the calibration parameters.

STRIPS

In STRIPS, all of the calibration parameters are multiplicative; i.e., growth estimates for a given forest type and species group are multiplied by the calibration parameter for that forest type and species group. If a model needs no adjustment (an

unlikely possibility), the calibration parameter will be 1; i.e., the “null value” of the parameter is 1. The calibration parameters are re-estimated until the average difference between the projected plot volumes and the actual plot volume is 0 for each species group and forest type. Table 3 shows the values of the STRIPS calibration parameters for Louisiana 1984-1991. The values typically are quite different from 1. The values range from 0.1 (the minimum value allowed) for cypress growth on oak-pine plots to 4.0 (the maximum value allowed) for cypress growth on oak-hickory plots. The calibration parameter values are greater than one more often than they are less than one.

Table 3.—STRIPS model calibration parameters

Forest Type	Pine	Cypress	Soft Hardwood	Hard Hardwood
Longleaf/Slash Pine	1.47	n.a.	3.31	3.40
Loblolly/Shortleaf Pine	1.64	2.00	2.01	1.54
Oak-Pine	1.45	0.10	1.75	1.41
Oak-Hickory	0.57	4.00	0.99	1.19
Oak-Gum-Cypress	1.12	1.37	1.08	0.74
Elm-Ash-Cottonwood	n.a.	0.83	0.65	1.03

RXWRITE.

The RXWRITE growth model uses 5 calibration parameters for each of 56 species groups (a total of 280 parameters). Each of the height, diameter and crown ratio models are adjusted with one parameter. Two parameters are used to adjust the mortality model. The height growth and diameter growth model calibration parameters are multiplicative. The calibration parameters for those models are re-estimated until the average height and diameter prediction errors, for trees where a matching record exists in both the projected and the actual tree lists, are zero. Unlike the predicted height or diameter growth, the change in the crown ratio may be either positive or negative. In this case, a multiplicative calibration parameter will have unpredictable results — increasing both positive and negative changes. Thus, an additive calibration parameter was used with the RXWRITE crown ratio change model. A constant (positive or negative) value is added to each predicted crown ratio change. In calibrating the crown ratio change model, the objective was to eliminate the average difference between the predicted crown ratio and the actual crown ratio for tree records found in both the projected and actual tree lists.

Predicted mortality was calibrated using a multiplicative calibration parameter that is a negative exponential function of diameter. This calibration parameter was actually multiplied by the probability of survival, rather than the probability of mortality. The resulting probability of survival can potentially be greater than one. This partially compensates for the lack of an ingrowth model in the system. The tree expansion factor (the number of trees per acre represented by the tree record) is multiplied by this probability of surviving. Equation (6) illustrates this, showing how the tree expansion factor for tree *i* in period *t+1* is calculated:

$$TrExp_{t+1,\,i} = k\, e^{-a\,DBH} \times [1 - P(mort)_{t,\,i}] \times TrExp_{t,\,i}$$

Here, *k* and *a* are the calibration parameters for the mortality function. (Note: *k* can be considered the intercept and *a* the slope of the negative exponential.) *P(mort)* is the probability of mortality for the tree, given by Equation (5). The objective

in searching for values of *k* and *a* for Equation (6) is to match the quadratic mean diameter and the number of trees for all trees in a given calibration species group in the set of plots used for calibration.[4]

As mentioned earlier, 56 species groups were used for calibration. This was done to try to correct for possible mis-aggregation of species in the 18 species groups used for statistical model development. In retrospect, 56 species groups was probably too many, but time limitations prevented us from re-doing the calibration with a smaller number of calibration species groups. Space limits prevent reporting all of the calibration parameter values calculated for the RXWRITE model. Table 4 summarizes the calibration parameters for RXWRITE by giving average parameter values for the four major species groups recognized by STRIPS.

Table 4.—Average calibration parameters for RXWRITE for four species groups

Model	Pine	Cypress	Soft Hardwood	Hard Hardwood
Ht	1.077	2.511	0.931	0.994
DBH	1.196	2.500	1.517	2.209
CR	0.0005	0.0104	-0.0010	0.0083
Int. (*k*)	1.219	1.208	1.235	1.377
Slope (*a*)	0.0168	0.0084	0.0229	0.0268

MODEL COMPARISON

After the models were calibrated, they were compared to determine which one best predicted the actual volume on each plot. To make this comparison, the standard deviation of the difference between the projected volume in each species group was calculated for each forest type. Let

$$Err_{sg,p} = \hat{V}ol_{sg,p} - Vol_{sg,p}$$

where $Err_{sg,p}$ = the prediction error for species group *sg* on plot *p*,
$Vol_{sg,p}$ = the observed (no hat) or projected (^) volume in species group *sg* on plot *p*.

A model produces unbiased volume projections if the expected value of $Err_{sg,p}$ is zero. Note that the STRIPS model was specifically calibrated so that the average value of $Err_{sg,p}$ is zero for the set of calibration plots, but there is no guarantee that this will be the case with the RXWRITE model. One would hope that if, on average, the number of trees on each plot is correct and the quadratic mean diameter is correct, then the average volume prediction would be nearly unbiased. The average prediction error for the RXWRITE model is reported in the tables below, and the reader can judge whether this was the case. A model does a better job of projecting plot volume when the standard deviation of $Err_{sg,p}$ is smaller.

Tables 5 through 8 show the results of this evaluation of the different models by forest type for each of the four species groups recognized by RXWRITE. For RXWRITE, the average error (bias) is also reported. The tables also show the average volumes per acre for each species group on each forest type. This value provides a reference against which the relative size of the average deviations can be judged. In almost every case, the STRIPS growth model outperformed the RXWRITE growth model. Typically, the average RXWRITE errors are 50 to 100 percent larger than the average STRIPS

[4] Note the difference between the set of trees used to calibrate the height, diameter and crown ratio models and the set used to calibrate the mortality model. For the first three models, only trees with a matching record in both the projected and the observed tree lists are used, and each tree record is given a weight of 1. All trees are used for calibrating the mortality model, and they are weighted according to their expansion factors.

errors. In spite of the general superiority of the STRIPS model relative to the RxWrite model, the average prediction errors of the STRIPS model are still uncomfortably high. This probably reflects the inherent poor quality, or noisiness, of the FIA data, and the problems of using these data for growth model development.

CONCLUSIONS

To the best of my knowledge, this is the first time a tree-level growth model and a plot-level (or stand-level) growth model have been developed and compared against each other using the same data set. In general, the fit statistics of the models were not good; r-squares were low. This is partly explained by the unavailability or poor quality of variables generally considered useful — some would say essential — for predicting growth and yield: age and site quality. In addition, FIA data are inherently noisy, representing a variety of very heterogeneous conditions. The uncalibrated models showed considerable bias, even when used to project the very data set with which they were developed. The calibration procedures applied with these models also are a unique feature of this study. The calibration parameters diverged considerably from their null values, reflecting the biases in the uncalibrated models. The final measures of the model errors showed that neither model predicts the future volume of FIA plots particularly well. All of this highlights the difficulties of projecting the state of FIA plots.

The results clearly demonstrate that the simpler plot-level model is better than the more complex tree level model. Of course, the comparison was made on the basis of the plot attributes that STRIPS is designed to project. Also, the STRIPS growth had the advantage of being fitted to data from two survey cycles (1974-1984 and 1984-1991), while the RxWrite model was fitted with data from only one survey cycle (1984-1991). In any case, the tree level model developed for RxWrite provides a wealth of additional detail in the projected plot data than the plot-level STRIPS model. If this additional detail is needed, then the tree-level model is obviously superior, in spite of its problems.

The results of this study likely will not surprise too many readers. They bear out the general principal that simpler models perform better than more complex models, and that additional complexity is generally only useful in a model if the additional detail provided by the model addresses some critical aspect of the process being modeled.

Table 5.—Model comparison for pine (all values are cu. ft. per acre)

Forest Type	Average Volume per Acre	STRIPS Standard Deviation	RxWrite Average Error	RxWrite Standard Deviation
Longleaf/Slash Pine	1757.44	593.29	104.78	771.2
Loblolly/Shortleaf Pine	2027.89	620.14	-3.08	806.28
Oak-Pine	1138.83	467.21	-155.88	711.91
Oak-Hickory	345.37	287.41	39.37	506.36
Oak-Gum-Cypress	33.19	62.74	-8.57	117.56

Table 6. Model comparison for cypress (all values are cu. ft. per acre)

Forest Type	Average Volume per Acre	STRIPS Standard Deviation	RxWrite Average Error	RxWrite Standard Deviation
Loblolly/Shortleaf Pine	3.19	15.99	-1.29	10.7
Oak-Pine	8.04	10.14	1.79	24.37
Oak-Hickory	39.08	127.24	2.34	69.93
Oak-Gum-Cypress	438.4	319.51	4.47	388.44
Elm-Ash-Cottonwood	137.85	76.01	-15.43	121.96

Table 7. Model comparison for soft hardwood (all values are cu. ft. per acre)

Forest Type	Average Volume per Acre	STRIPS Standard Deviation	RxWrite Average Error	RxWrite Standard Deviation
Longleaf/Slash Pine	114.51	213.78	-22.84	117.64
Loblolly/Shortleaf Pine	231.68	156.33	42.22	192.71
Oak-Pine	482.63	176.62	21.27	301.64
Oak-Hickory	572.79	199.26	-28.90	300.37
Oak-Gum-Cypress	1,028.95	362.30	-111.67	551.80
Elm-Ash-Cottonwood	1,502.88	618.21	133.27	1,045.78

Table 8.—Model comparison for hard hardwood (all values are cu. ft. per acre)

Forest Type	Average Volume per Acre	STRIPS Standard Deviation	RxWrite Average Error	RxWrite Standard Deviation
Longleaf/Slash Pine	66.38	87.44	-42.39	102.35
Loblolly/Shortleaf Pine	301.76	211.44	9.81	237.04
Oak-Pine	702.23	245.72	-24.75	499.49
Oak-Hickory	1,034.64	308.92	-110.90	638.96
Oak-Gum-Cypress	837.62	285.22	-66.64	735.95
Elm-Ash-Cottonwood	306.77	278.07	-199.82	423.95

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