OPTIMIZING STAND LEVEL FOREST HARVESTING USING A TRADE OFF OBJECTIVE FUNCTION

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ABSTRACT.—We have developed a tabu search method to optimize the stand level harvest decision over intermediate time frames (10-30 years) within the context of a longer term hierarchical plan. In this paper, we focus on the objective function for our model. We are interested in the trade off between two main issues, one the cost of building roads to access the resource and the other the loss of forest productivity when stands are harvested too early or too late. We develop a biological productivity loss function and show how to examine the trade offs between the cost of roads and the productivity losses.

INTRODUCTION

In this paper, we address the problem of optimizing the stand harvest schedule for medium-range tactical planning, while meeting volume requirements in each period as specified by a previously executed strategic planning process. We assume that accessing stands in the forested area will require some road building, and that, consequently, a network of potential road links has been designed to cover the area. Although many tactical level objectives are worth considering, we will concentrate on the minimization of two costs. The first is the negative impact on long-term forest productivity caused by harvesting stands either too early or too late. The second is the cost of building roads to access the stands to be harvested.

These cost factors are in opposition, since improving the timing of harvests can only be done by increasing the investment in road building. We do not assume, however, that the decision-maker has fixed a road building budget or a tolerance level for lost productivity. Instead, we produce a spectrum of solutions to the scheduling problem which range from those with very little road building to those where a high investment in road construction is required. By then recording the non-dominated solutions, we produce an efficient frontier or trade off curve which clearly presents the trade off effect over the range of road budget decisions which are available.

Our model is constrained to meet maximum opening size and adjacency delay constraints. We do not assume that the adjacency delay period is equal to one planning period. The spatial decision unit is the stand, avoiding pre-blocking of stands. We use a Tabu Search metaheuristic to solve the model.

This paper is organized as follows. In section 1 we describe the planning problem and the model. In section 2 we describe the basic solution philosophy and technical issues in dealing with adjacency constraints and the road network. In section 3 we present some results, which were obtained from a forest coverage of a region of Cumberland County, Nova Scotia. Section 4 concludes with some conclusions and discussion.

THE PLANNING PROBLEM

This problem is to optimize decisions made at the tactical level of forest planning. The tactical planning problem is considered in the context of a hierarchical forest planning system [Weintraub and Davis 1996, Weintraub and Cholaky 1991, Gunn and Rai 1987], where the strategic planning process has been previously executed. The strategic planning exercise has used a long planning horizon and, amongst other goals, has enforced the requirement that production levels be consistent with sustained yield. One result of this process is the

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recommendation of achievable levels of sustainable wood harvest over some longer planning horizon Then, the tactical planning problem is to produce a spatially explicit schedule of stand harvests and road building. This schedule must meet the strategic goals on volumes. It is further constrained to meet maximum opening size restrictions and adjacency delay constraints. In the IRPM formulation [Kirby et al 1980], as in many other models in the literature, the objective function is to maximize net present value accruing from the periodic flows of wood products. We take a different approach. We assume that harvest levels, as determined by the strategic planning process, have already been maximized in the aspatial model subject to suitable constraints. Thus, achievable levels of production are already optimized. In our model, we restrict our attention to minimizing negative impacts on forest productivity due to timing of harvests and the costs of road construction, and constrain the model to meet the recommended production levels for each period. The harvest schedule is also constrained to meet maximum opening size and adjacency delay regulations on clearcuts. This framework for our model is illustrated in Figure 1.

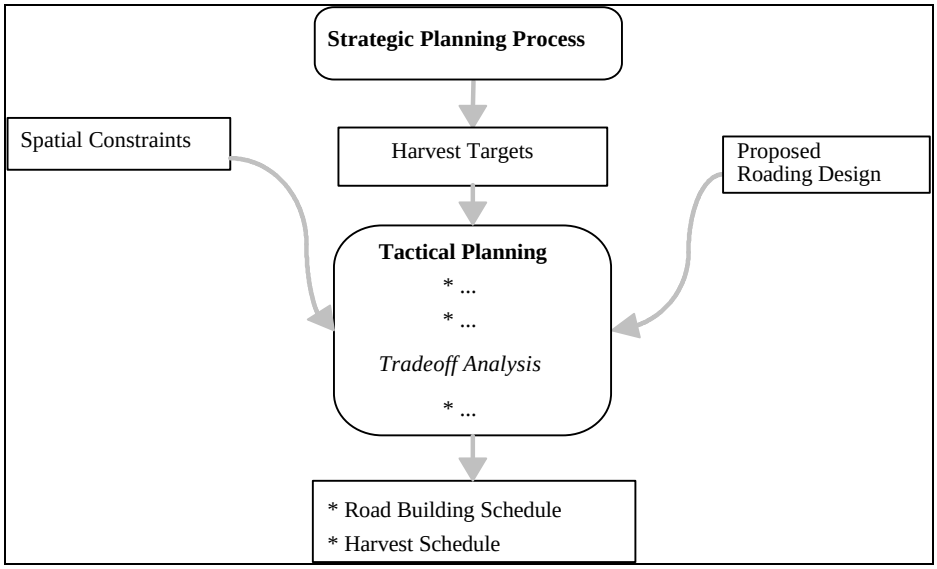

Figure 1.—Model framework

Volume Flow Constraints

Let TV, PV_j and dTV, dPV_j be the total and periodic volume targets and their allowed deviations, V_{sj} the volume flow from harvesting stand s in period j, J the number of planning periods and x_{sj} an indicator variable for harvesting stand s in period j. Then, the constraints on total and periodic volume flows are:

(1) $$\left|\sum_j \sum_s V_{sj} x_{sj} - TV\right| \le dTV, \; and \; \left|\sum_s V_{sj} x_{sj} - PV_j\right| \le dPV_j, \; j = 1, \ldots, J$$

We will use the S- vector $\boldsymbol{x}$ to indicate a given harvest solution, where each element x_s is the harvest period assigned to stand s. The assignment $\boldsymbol{x_s} = 0$ indicates that the stand was clearcut in the previous planning period, and $\boldsymbol{x_s} = J+1$ means that the decision is to defer harvest decisions to the future.

Spatial and Temporal Constraints on Harvest

The adjacency delay time or green-up period is defined in integer multiples of the planning period units. For our problem, planning periods are of duration 5 years and green-up or adjacency delay is 10 years [Province of Nova Scotia 1990]. The elementary spatial decision unit is the stand, which is generally much smaller in area than the maximum opening size. We have evolved the following definitions so that the adjacency delay and maximum opening size constraints can be effectively specified using adjacency graphs.

Definition 1.1 The *forest adjacency graph* or forest graph *FG=(S, E)* is a set of nodes *S*, one for each stand, and an edge *e* in *E* for each pair of stands which share a common boundary (or are a minimum distance apart).
Definition 1.2 The *adjacency delay time ADT* is the number of planning periods required for green-up on adjacent openings.
Definition 1.3 $S_p(x) = \{s \mid x_s \in [p - ADT + 1, p]\}$, $p = 1, \ldots, J$, the set of stands scheduled within one adjacency delay "window".
Definition 1.4 $E_p(x)$ is the set of edges of *E* with both end-nodes in $S_p(x)$.
Definition 1.5 $F_p(x) = (S_p(x), E_p(x))$ is the sub-graph of *FG* induced by $S_p(x)$ and $E_p(x)$.
Definition 1.6 An *opening* of period *p* $O_k^p(x)$ is a connected sub-graph of $F_p(x)$.
Figure 2 illustrates these openings for an *ADT* of two periods. Maximum opening size and adjacency delay requirements are then simultaneously enforced by requiring that each opening be of size less than or equal to *maxopen.*

(2) $$\sum_{s \in O_k^p(x)} a_s \le \text{maxopen}, \quad k = 1, \ldots, K^p(x), \; p = 1, \ldots, J; \; a_s \text{ is the area of stand s.}$$

Since the openings are dependent on a given harvest schedule **x**, there is no convenient method of specifying these constraints in an integer programming formulation. Enumeration of all potential openings which could cause a *maxopen* violation is possible, but not practical since the number of possible openings is exponential in the number of stands. However, the computational complexity of graph searches to calculate these openings is rather trivial in this application, since the forest adjacency graph is sparse. Thus, although this is not a useful model for integer programming methods, it will prove to be workable for our heuristic search methods.

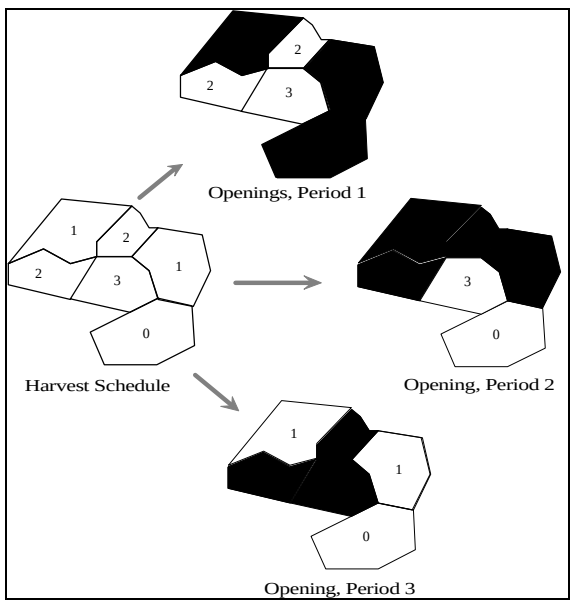

Figure 2.—Multiperiod openings

Model Structure

In this problem the decisions are which stands to harvest, and when. We penalize the choice to harvest a stand at a period other than its peak productivity period using the Lost Volume penalty function *LV(s, j)*. The cost to build roads is determined by solving an induced road network design problem. These two cost factors are described in the following sections.

Lost Volume Penalty Function

The lost volume penalty function estimates productivity losses which are incurred by scheduling stands for harvest too early or too late. The mean annual increment, *mai(g)*, of a stand at age *g* is the average wood volume increase per unit area per year at full stocking. For simplicity we will refer to *mai(g)* as *mai*. Mai depends on the existing cover type, age and site capability of the stand [Gunn 1994, Province of Nova Scotia 1993]. Figure 3 shows mai curves for stands of site index 3 - 8 for Nova Scotia softwoods (higher numbered site index indicates better growth potential). If the biological production potential of the forest is to be maximized, then stands should be scheduled for harvest when they have reached their maximum *mai*. The lost volume penalty induced from assigning stand *s* to period j then is a function of the stand size, age, stocking, and mai and the current growth state of the stand. For *growing* stands the contribution to lost volume from assigning stand *s* to harvest in period *j* is

(3a) $$LV_{growing}(si,j) = |mai_\max(si) - mai(si,j)| \times a_s \times age_s(j) \times stocking,$$

where *si* is the stand's site capability index, *j* is the harvest period, *age(j)* is the age at period *j*, a_S the acreage of stand *s* and *stocking* the percentage of full stocking on stand *s*.

For stands which have a significant amount of dead timber, we define a different lost volume function. The presumption is that the stand will resume normal growth after the inhibiting deadwood is removed and some remedial silvicultural action taken. Consequently, for *dead* stands, we penalize for each period that we wait to intervene.

(3b) $$LV_{dead}(si,j) = \begin{cases} 0 & \text{if } j = 1 \\ \text{mai_max(si)} \times (j-1) \times a_s \times \text{stocking} & \text{if } j \in [2, J+1] \end{cases}$$

Thus, if the stand is cut in the first planning period, no penalty is applied, since this is the best action that can be taken. Otherwise, the delay to future periods is penalized by the expected future growth at maximum mai. The Lost Volume function is illustrated in figure 4, where stand *s* with the indicated *mai* curve is at age class 7 in period 1 of the planning horizon. The penalty for early harvest at period 1 and the penalty for late harvest at period 5 is illustrated in Figure 4(a). If stand *s* were dead, the penalty to harvest (or treat) in period 4 is shown in Figure 4(b).

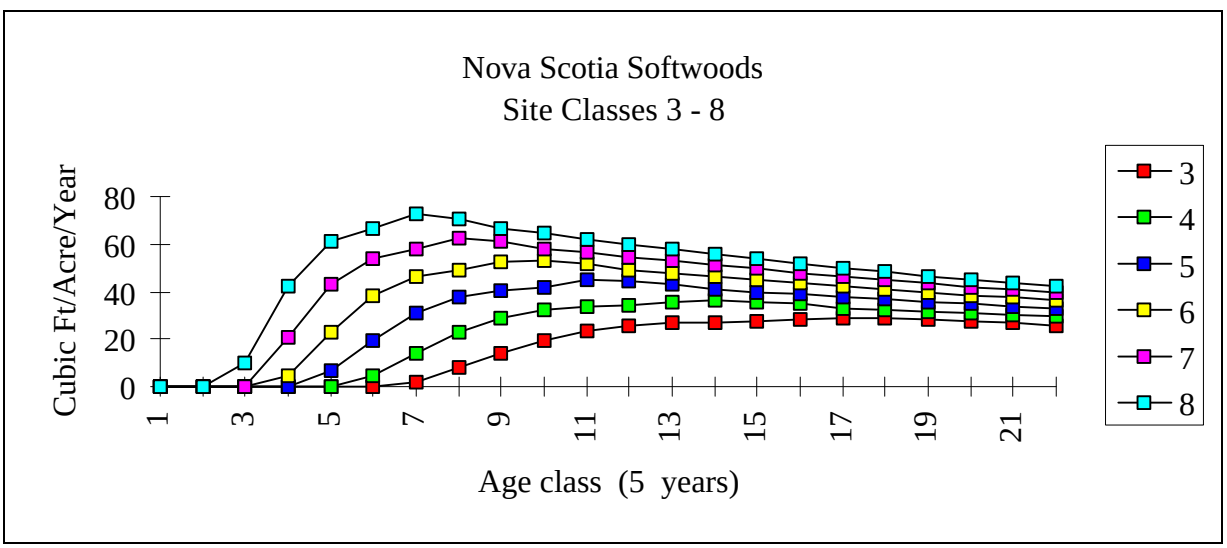

Figure 3.—Mean annual increments

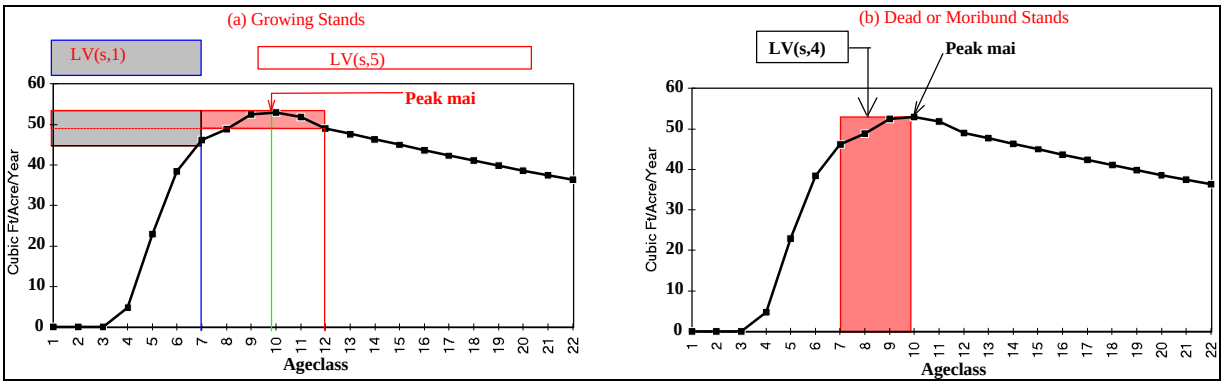

Figure 4.—Lost volume function

Road Network

This network problem is based on an existing set of road links which cover the forest planning area. Figure 5(a) shows how a typical potential road network relates to stands and existing main roads. Multiple access points and cycles in the network are allowed, and each stand is accessed by one of the road links, or by a main road. For each link *r*, we associate the set S_r, consisting of all stands which require constriction of link r for access. Then, each harvest schedule ***x*** gives rise to a road network design problem, ***RNP(x)***.

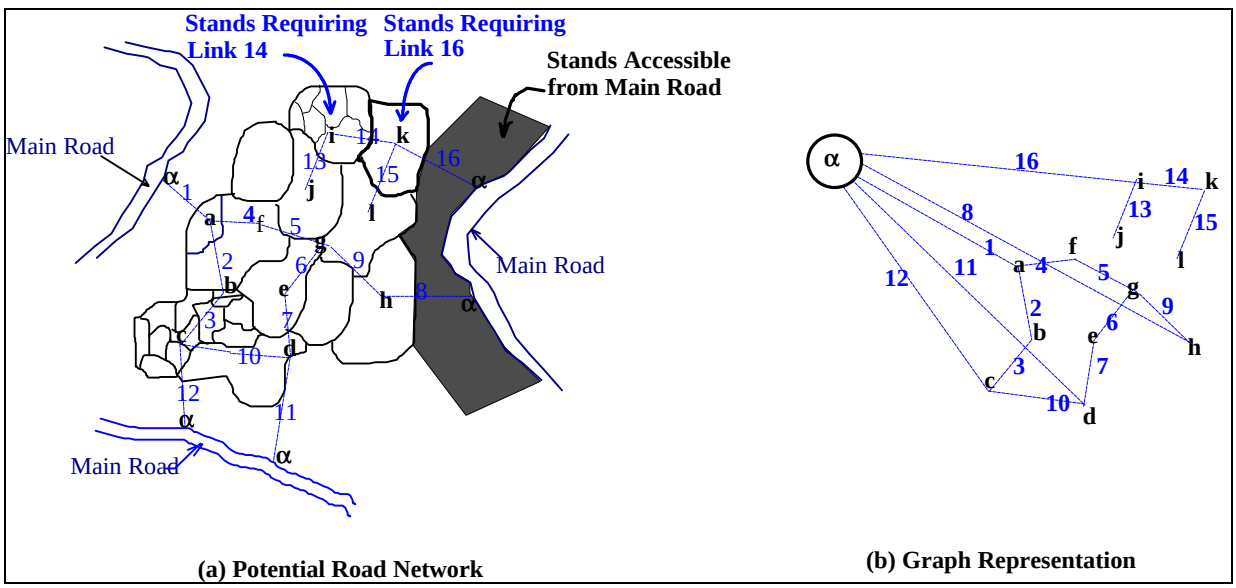

Figure 5. Road Network

The network is represented as a weighted graph *RG = (V, R, W)*, where the edges *R* are road links and each weight w_r is the cost to build link *r*. Vertices *V* are end nodes of the links, which are indexed in any unique fashion, with the proviso that only one node index is used to indicate that a link accesses a main road. That is, all existing main roads are "collapsed" into one node α.

For any harvest schedule ***x***, there is a set of required links $RL = \{r \in R | x_s \in [1, J], s \in S_r\}$. Each link in *RL* has an earliest required construction period $cp(r) = \min_{s \in S_r}\{x_s\}$. A feasible solution to ***RNP(x)*** is a directed connected subgraph *S* (*V'*, *R'*) of *RG* such that there is a directed path in *S*, $P_r = \{r, r_{n_1}, r_{n_2}, \ldots, r_{n_k}\}$ from each required link ***r***

in *RL* to α. That is, the ending link in this path, r_{n_k}, has one node equal to a. Each link in each path is assigned a construction period, a mapping *W* from *R'* into [*1*, *J*] such that $\Omega(r_{n_k}) \le \Omega(r_{n_{k-1}}) \le ..\Omega(r_{n_1}) \le \Omega(r)$. An optimal solution is one of minimum weight, i.e. one that minimizes the discounted costs of road building $\sum_{r|\Omega(r)\le J} \frac{w_r}{(1+d)^{\Omega(r)}}$ (*d* is the discount rate). If we restrict the problem to one period, or omit the discounting, it can be shown that it is equivalent to the Steiner Tree Problem on Graphs. Some special cases of STP are solvable in polynomial time, but in general the Steiner tree problem is NP-complete.(Garey and Johnson 1979). Accordingly, we developed a heuristic method to solve ***RNP(x)*** (Richards 1997).

Model Objective Function

The model objective function is a parameterized sum of the lost volume penalty and the road construction cost. Using indicator variables $\boldsymbol{x_{sj}}$ and $\boldsymbol{z_{rj}}$ for harvesting stands and building roads in period j, the objective is

$$z(r) = \sum_{j=1}^{J+1}\sum_{s=1}^{S} LV(s,j)x_{sj} + r\sum_{j=1}^{J}\sum_{r=1}^{R} w_{rj}z_{rj} \qquad (4)$$

and the problem is to minimize ***z(r),*** subject to constraints (1) and (2) and road network feasibility. The parameter, ρ, is used to drive the solution algorithm to produce a spectrum of solutions with differing proportions of road cost to lost volume. After solving the model for a range of values of ρ, we have the range of solutions to produce the trade off curves.

METHOD OF SOLUTION

We have chosen to use a Tabu Search strategy to solve this problem. Tabu Search (TS) belongs to the *metaheuristic-heuristic* class of algorithms, where local optimization (local neighborhood search) is enhanced with strategies which enable the search process to go beyond local minima. The seminal ideas behind tabu search were first developed by Fred Glover (Glover 1986), and independently by Pierre Hansen (Hansen 1986). Tabu search is a history-based or memory-based strategy; information from previous phases of the search is used to direct future phases. We refer the reader to (Glover 1992) for a complete description of the TS method and applications, and to (Richards 1997) for a complete description of the algorithms used in this work. Here, we will restrict the description to an overview of some of the technical aspects of solving this problem using TS. The following is an outline of the algorithm.

Tabu Search Algorithm

1. Generate a feasible starting solution *x* using Monte Carlo methods, and calculate the road network.
2. Neighborhood search: Evaluate each non-null move $x_s \leftarrow j, j \in [1, J+1]$ using the evaluation function $z'(x,r) = LV + rRC + aDev_total + bDev_per + gDev_maxopen$, where *LV* is the total loss penalty, *RC* the road cost and terms Dev_total, Dev_per, Dev_maxopen are penalty terms for infeasible moves. The coefficients α, β, and γ are dynamically assigned throughout the search process to effect a strategic oscillation through the boundaries of the feasible region (Glover 1992, Gendreau, Hertz and Laporte 1991).
3. Accept the best non-Tabu move, implement the move and update the Tabu List Structures
4. If *z(r)=z'(r)* (i.e. the solution is feasible) is less than any found so far, save the solution.
5. If the stopping rule is met, stop.
6. If diversification is required, execute diversification routine.
7. Go to neighborhood search.

Some comments on the technical aspects of neighborhood search and move implementation are in order. For each move evaluation or implementation we do a depth first graph search (Sedgewick 1983; Lockwood and Moore 1992) to calculate opening sizes and any ensuing violations of the adjacency constraints. Stand assignments induce a road network design problem which is solved using a path heuristic. Since this problem ***RNP(x)*** is *NP-hard*, we

use a heuristic procedure to solve this problem. The heuristic is based on a modification of Prim's Shortest Path Spanning tree algorithm (Sedgewick 1983). This algorithm is fully described in (Richards 1997). Tabu tenure and the requirement for diversification is determined by a reactive mechanism which measures cycling of the search trajectory (Battiti and Tecchioli 1995). See (Richards 1997) for further details of the tabu search method.

EMPIRICAL RESULTS

We tested the method on an area of 1039 stands, with a proposed road network of 125 links. The total harvest target over four five-year planning periods was two million solid cubic feet, equally distributed between periods. Total percentage deviation allowed was 2 percent, periodic 5 percent. In this region, the maximum opening size is 125 acres and adjacency delay 10 years. The cost to build one km of road is $5000 and is discounted at 10 percent per year.

Solutions

Figures 5 and 6 show solutions for ρ equal to 1.0 and 10.0. Figure 7 summarizes the number of kilometers of road to be built in each period. Figure 8 shows that it is possible to construct feasible solutions harvesting stands which are accessed by the existing main roads. The cost in lost productivity is, however, extremely high, approximately 1 million cubic feet of lost volume.

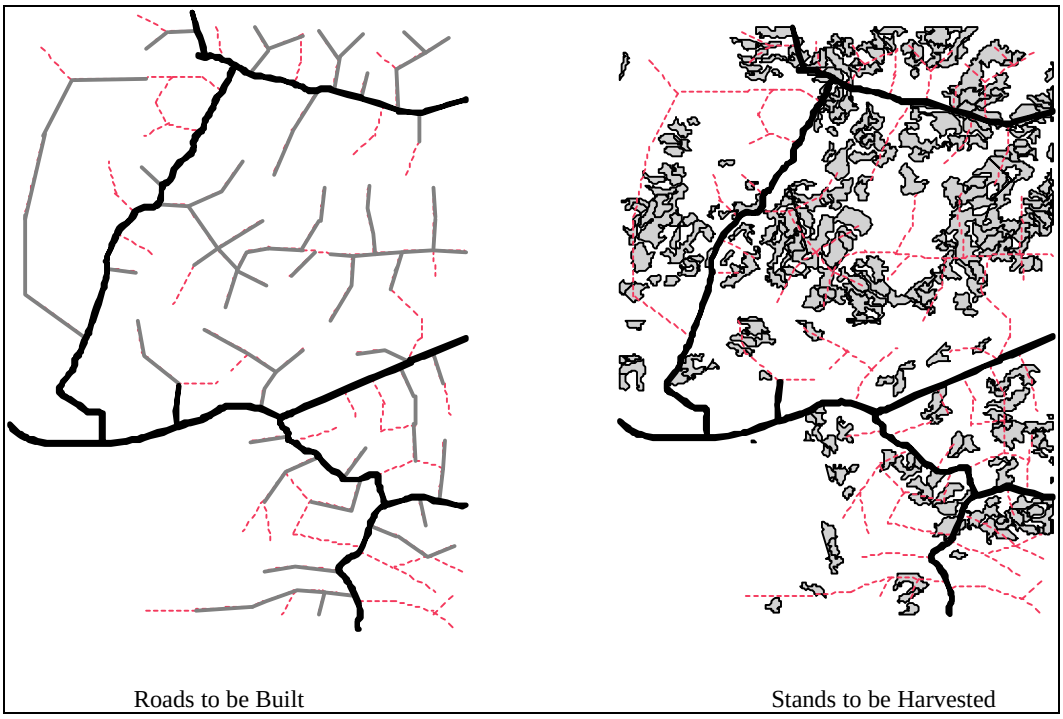

Figure 5.— Solution for ρ = 1.0. Solid lines show roads to be built; dashed lines the unbuilt links and heavy lines the existing main roads. Roading Cost is $99,132 and lost volume penalty 306,526 cu. ft.

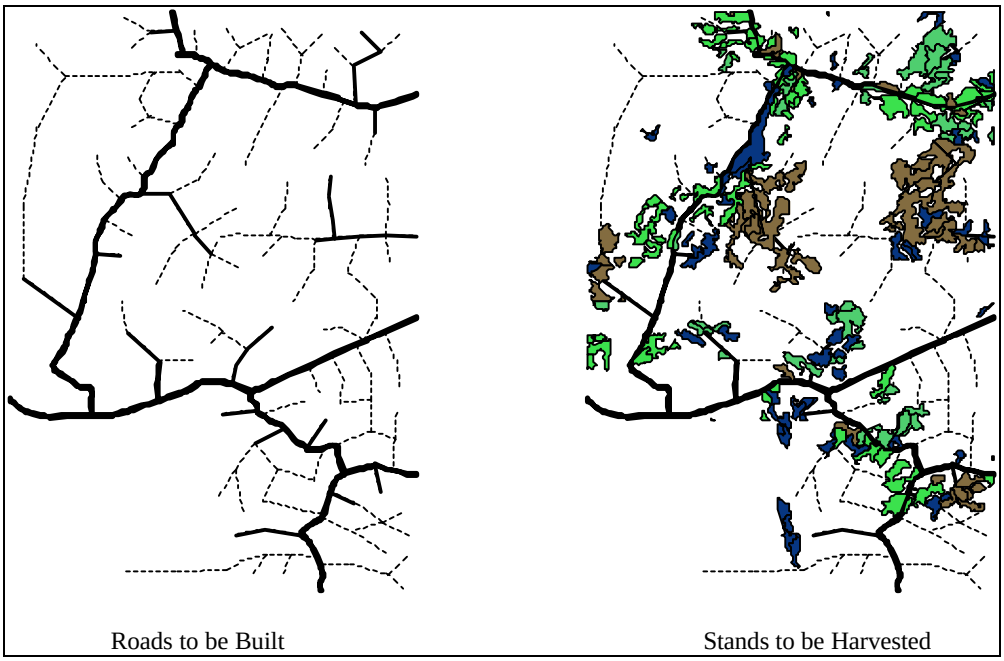

Figure 6.—Solution for ρ = 10.0. Roading cost is $24,840 and lost volume 563,444 cu. ft.

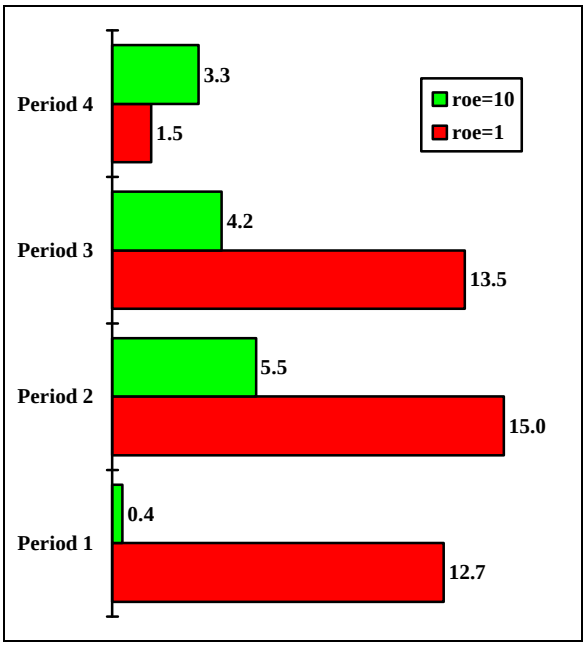

Figure 7.—Km of roads built by period.

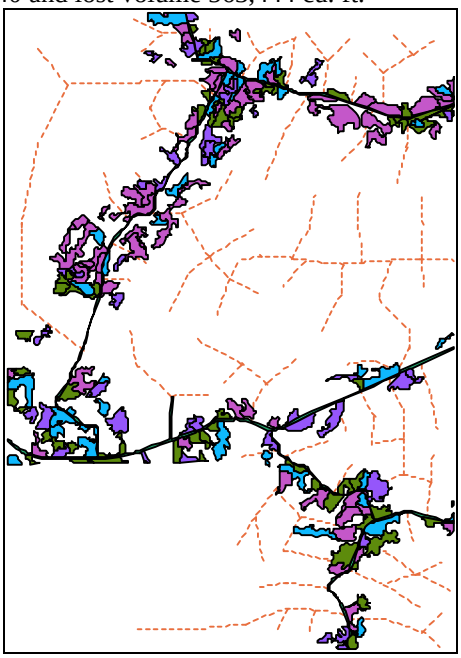

Figure 8.—Solution for ρ = 20.0 No roads.

Trade off Curves

The trade off curve is the efficiency frontier for the two attributes, minimum road cost and minimum lost productivity. Figure 9 shows the curve produced from this dataset. At the extremes, we see that it "costs" 950,000 cubic feet of lost volume to spend nothing on road building. At the other extreme, we can spend $175,000 on road building (at $5,000 per km) to drive the lost volume to approx. 300,000. More importantly, we can see that by increasing investment in road building to $40,000, we can reduce the lost volume to approximately 425,000 cu. ft.,

a reduction of 50 percent. Furthermore, the curve shows us that there is very little point in increasing road spending to more than $80,000 since the returns on reducing lost volume are small.

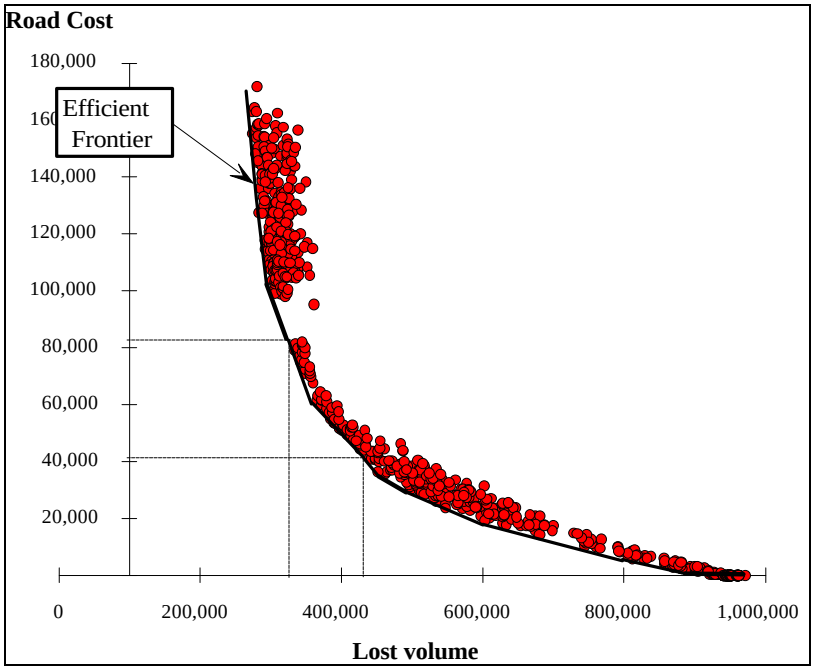

Figure 9.—Trade off curve

DISCUSSION AND CONCLUSIONS

We have presented a model and a workable solution method to perform trade off analysis in the context of tactical forest planning. We believe that the trade off analysis provides valuable information to decision-makers and should improve the planning process. We have also produced a Tabu Search algorithm which is applicable to this problem. Our modeling of adjacency delay and maximum opening size restriction using graphs and multiperiod openings does not require pre-blocking of stands, and does not require the planning period length to be equal to the green-up period. The graph search calculations are practicable in sparse forest adjacency graphs. In our model, we have considered only adjacency and production constraints. The method does not preclude the inclusion of other constraints, although some work would be necessary to refine the neighborhood search and diversification strategies in the Tabu Search algorithm.

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