Use of Three-Point Taper Systems in Timber Cruising

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Abstract.—Tree volumes and profiles are often estimated as functions of total height and DBH. Alternative estimators include form-class methods, importance sampling, the centroid method, and multi-point profile (taper) estimation systems; all of these require some measurement or estimate of upper stem diameters. The multi-point profile system discussed here allows for upper stem measurements at any heights and constructs a smooth profile prediction that passes through all measurement points. Results from a cruise sample of trees can be summarized to allow for improved profile predictions for the non-sampled trees, which lack upper stem measurements. Decisions must be made on which measurement heights to use, which trees to sample, and how to make the upper stem measurements. The driving forces in sample efficiency are measurement error, between-tree and between-stand variation in form, and within-tree correlations at differing heights. Results from several sources are brought together to illustrate the magnitude of the variances; the focus here is on how to reduce the between-stand errors associated with taper and bark thickness. Suggestions are offered for efficient use of three-point profile systems, addressing measurement techniques, sampling methods, and summarization procedures.

A common objective of timber cruising is to estimate volume and other characteristics for particular stands. Often this is accomplished without any direct measures of tree form or volume. Volumes are estimated for sample trees based on their species, diameter at breast height outside bark (DBH), and total height (HTOT); two common estimation systems are volume prediction equations and stem profile prediction systems. The latter typically estimate diameter inside bark at any height (h). The usual purpose of profile prediction is as an intermediate step in predicting total volume or merchantable volume. A limitation of both taper and volume prediction systems is that for particular stands, there may be significant errors; the volume equation or taper function will be incorrect for any given stand, and the mean error may be anywhere from zero to 10 percent or more. The reasons for the errors might have to do with how the volume or taper function was fit; other, more benign possibilities include non-average tree form due to management practices, genetics, climate, or other unidentified factors. Such form differences may be major contributors to observed differences between preharvest cruises and cutout tallies.

There are several methods to overcome the above limitation. These include three-point volume equations (McTague 1992), several within-stem sampling approaches (Wiant *et al.* 1996), and multi-point profile estimation (Rustagi and Loveless 1991, Flewelling 1993a).

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THREE-POINT TAPER PREDICTION SYSTEMS

The taper applications described here have all been developed as "multi-point systems" having as input: DBH, total height, and one or more upper stem diameter measurements. The user decides how many measured upper stem points are to be used in an application. In the application described here, only one upper stem measurement per sample tree is used. Some multi-point systems require the user to measure upper stem diameter at particular heights. The system developed by Flewelling (1993a) allows the user to choose the height or heights at which extra measurements are taken. That system is a generalization of a two-point prediction system. The general requirements for this formulation are:

1. A profile prediction system that predicts diameter inside bark (d) at any height (h) for a stem with measured diameter at breast height (DBH) and measured total height (HTOT).

2. An estimate of the variance (var) of the prediction errors (the differences between actual and estimated diameters) as a function of tree size and height (DBH, HTOT).

3. A bark thickness prediction at breast height—or a plan to measure or otherwise estimate on every tree.

4. A formula to estimate the correlation (q) between prediction errors at any two heights on the stem.

5. A bark thickness prediction at heights other than breast height (BH).

Examples of relationships (1) to (3) are in Flewelling and Raynes (1993); (4) and (5), and the underlying theory of multi-point estimation are given by Flewelling (1993a). The essential theory is reviewed here.

Suppose that at some predetermined height on the stem (h_i), the true diameter inside bark (d_i) could be obtained; actually diameter outside bark is measured, and d_i is estimated as diameter outside bark minus an estimated double bark thickness. A standardized error (or disturbance z) can be computed as:

$$z_i = (d_i - \hat{d}_i) / \sqrt{\text{var}_i} \qquad (1)$$

where $\hat{d}_i$ is from the original stem form prediction system (a "two-point" system), d_i is the observed or measured diameter, and var_i is the estimated variance. Based on this observed error, we wish to make a revised prediction of diameter inside bark at every height on the stem. For height h_j, the appropriate estimator is based on the original estimator plus the conditional expectation of the error:

$$\hat{d}'_j = \hat{d}_j + q_{ij} \times z_i \times \sqrt{\text{var}_j} \qquad (2)$$

where q_{ij} is the estimated correlation. A sufficient rationale for the above equations is the assumption that among trees of a given size (DBH, HTOT), the diameters at any two predetermined heights have a bivariate normal distribution.

A typical cruising application of the three-point system is to measure DBH and total height on a sample of trees and to measure one upper stem diameter at a predetermined height on all or a subsample of these. The predetermined height may be a constant, or a relative height. For example, we refer to the 30 percent height as being BH + 0.3 x (HTOT - BH), where BH is breast height; this is a good height at which to measure *dob* for an improved estimate of total stem volume. Alternatively, any other relative height or a fixed height above the ground could be used. From each upper stem *dob* measurement, a *dib* estimate is inferred and converted to a z value (the "standardized" error), by means of equation 1. Thus, z is a measure of stem form, and mean z is a mean measure of form for a stand. Within a stand, the individual z values are regressed against DBH, or averaged. The average z, or the regression prediction of z, is presumed to apply to the trees that did not have an upper stem dob measurement. Diameter predictions at all other heights are from equation 2. Thus, for a typical tree lacking upper stem measurements, the inputs to the stem form prediction system are DBH, HTOT, the stand mean of z and height for which the z value is assumed to apply.

In our experience, the trend of z versus DBH is usually flat. In such cases, we recommend that the mean z be used, without regard to statistical testing for non-zero means. It is common for individual stands to have mean z values that are significantly different from zero. If this were not a common occurrence, the presumption would be that no benefit was being derived from the measurement of upper stems: either form does not vary between stands, or the sampling and measurement errors are overwhelming any stand-to-stand differences in tree form.

INGY DATA AND MODEL

The Inland Growth and Yield Cooperative (INGY) sponsored a project in 1995-1996 to collect data for and develop profile prediction equations for commercial species found in Montana, Idaho, eastern Washington and Oregon, and in the southern interior of British Columbia. The data consisted of several large historical databases, and newly collected data for 10 species. The new data were obtained by following a designed tree selection procedure in stands in all areas except British Columbia. The protocol ensured that the trees for each species would come from a large number of stands spanning site/habitat conditions, that a wide range of trees sizes (DBH, HTOT) would be felled, and that a consistent measurement protocol would be followed. Diameter inside bark (*dib*) and outside bark (*dob*) measurements were taken at regular intervals along the stem. The sample sizes for the new data are reported in table 1. The older data records involve about 13,000 trees. All data were used in the development of the stem-profile models, but the new data were accorded higher weights such that they made up about 40 percent of the total weight.

The fitted stem profile model was a variant of that reported by Flewelling and Raynes (1993); it was adopted directly from an unpublished report that Flewelling prepared for an earlier co-operative study in western Oregon and Washington. The fitting methodology was a maximum likelihood method described by Flewelling (1993b); the error distribution was assumed to be lognormal with variances dependent upon height and tree size. The thickness of the bark at heights other than breast height was modeled as a function of tree size, height, and *dob*. Statistical tests showed that tree form for some species varied by geographic region. Accordingly, models for several of the species have one or two coefficients that vary with geographic region. The models are described in an unpublished report by Flewelling and Ernst; a USDA Forest Service publication is presently in a draft stage.

Table 1.—*Numbers of sample trees by species and by numbers of sample trees per stand; INGY data set. Thus the table describes the 986 trees specially collected for the INGY project.*

Species	Numbers of sample trees per stand				
	1	2-3	4-5	≥6	All
Douglas-fir	24	59	47	34	164
Western larch	23	56	35	19	133
Grand fir/ white fir	12	50	26	29	117
Ponderosa pine	10	36	40	48	134
Lodgepole pine	14	55	33	34	136
Western red cedar	7	17	17	0	41
Hemlock	7	24	17	45	93
White pine	6	13	5	0	24
Engelmann spruce	10	39	18	7	74
Subalpine fir	6	33	16	15	70
Total	119	382	254	231	986

DATA FOR VARIANCE MODEL IN VOLUME

As explained in the introduction, a major purpose in using multi-point taper systems is to reduce the errors in volume estimation that are due to between-stand variances. To study the effectiveness of the models for this purpose, we will define several volume error variables, estimate their variance (V), and partition that variance into between-stand and within-stand components. Furthermore, we propose to estimate how much of the between-stand variance can be eliminated by the use of upper stem *dob* measurements, how this is affected by measurement error and sample size, and how much further improvement could be made by measuring bark thickness at breast height. This is all done without relying on any of the statistical assumptions made during the fitting of the model.

As part of the fitting process, total inside-bark cubic volume (including top and stump) was calculated for each tree in the database. These volumes were computed as the integration of a preliminary fit for the profile equation, plus an integration of the errors in cross-sectional area. These volume calculations should be mainly free of the biases that can arise in the more common method of assuming that each log section is a particular geometric frustum. We refer to the resultant estimate of actual total tree volume as "VOLA." Predicted total tree volumes (VOLP) come from a numerical integration of the predicted profiles; interval lengths are generally 3 percent of total height; below breast height, shorter intervals are used.

The underlying stem-form predictions are always based on DBH, total height, and where appropriate, region. Additional volume predictions are made by including various combinations of input variables, which may include one upper stem dob and the measured bark thickness at breast height (DBTbh). We number the various prediction methods according to the combinations of input variables.

1. Base (DBH, total height, region)
2. Base plus dob at 10 percent relative height (dob10)
3. Base plus dob at 20 percent relative height (dob20)
4. Base plus dob at 30 percent relative height (dob30)
5. Base, plus DBTbh
6. Base plus dob at 10 percent relative height, plus DBTbh
7. Base plus dob at 20 percent relative height, plus DBTbh
8. Base plus dob at 30 percent relative height, plus DBTbh

For each of the eight estimation systems, VOLP is predicted, and an error is calculated as:

$$ERR = \ln(VOLA / VOLP)$$

where *ln* refers to the natural logarithm. In fitting volume equations, this variable is typically assumed to have a normal distribution with zero mean and constant variance. A general linear model estimates the variance components for ERR: within-stand variance (σ_{ws}^2), and the between-stand variance (σ_{bs}^2). Restricted maximum likelihood estimation is used. The data being used are the new INGY data except that trees from the first column in table 1 are omitted—the trees that lack other within-stand sample trees of the same species. Thus, a given stand is assumed to have some unknown mean error for each species; no relationship is assumed between two species in a common stand. The resultant estimates of variance for ERR are in table 2.

Table 2.—*Variance estimates for the errors in ln(volume)*

Prediction method	Between-stand	Within-stand
1 Base (DBH, Totht, region)	.00155	.00664
2 Base & dob10	.00120	.00357
3 Base & dob20	.00081	.00288
4 Base & dob30	.00081	.00234
5 Base & DBTbh	.00117	.00570
6 Base & dob10 & DBTbh	.00068	.00322
7 Base & dob20 & DBTbh	.00030	.00266
8 Base & dob30 & DBTbh	.00020	.00200

Before we discuss the interpretation of these results, one other result is needed: the relationship between an upper stem measured *dob* and predicted volume; this will allow us to address the effect of random measurement errors in *dob*. The underlying model is:

$$\ln(VOLP'/VOLP) = m \times \ln(dob'/dob)$$

where *dob* is the actual upper stem diameter, *dob'* is the corresponding measurement with introduced error, VOLP is the predicted volume calculated using *dob*, and VOLP' is the predicted volume calculated using *dob'*. The coefficient *m* is estimated as the mean, over all trees, of *ln*(VOLP'/VOLP) divided by *ln* (1.01), where VOLP is from the three-point prediction system using actual *dob*, and VOLP' is from the same prediction system, but assuming that dob is 1 percent greater than it actually is—thereby simulating a 1 percent measurement error. For relative heights 10 percent, 20 percent, and 30 percent, *m* is estimated as 1.262, 1.318, and 1.225, respectively. Simulations of other errors in *dob*, from -5 percent to +5 percent confirm that the relationship between ln(dob) and ln(VOLP) is almost linear. Although this sensitivity relationship is calculated at an aggregate level, it is assumed to be approximately correct at the individual tree level.

VARIANCE MODEL IN VOLUME

If in application every species in every stand had a sufficiently large sample of trees, the resultant between-stand errors in volume (by species) would be as indicated in table 2. Using only the base measurements, the variance in errors for *ln*(volume) would be 0.00155, corresponding to a between-stand standard deviation of about 3.9 percent. This is in addition to the normal sampling errors and reflects only the stand-to-stand uncertainty in stem form and bark thickness. In method 4, which requires measurements of dob at 30 percent relative height, the between-stand variance drops to .00081, corresponding to a standard deviation of 2.8 percent. Hence, measurements of *dob* at relative height 30 percent reduce the error variance (at the stand level) by almost half. A further reduction in variance occurs if bark thickness at breast height is also measured. The total error for a species' volume will, of course, be increased due to sampling errors in basal area and height; these sources of error are not addressed.

For finite sample sizes, the actual benefits of the additional measurements (*dob* at a relative height and/or DBTbh), will be less than that indicated by table 2. For example, if in a stand only one tree of a given species had an upper stem measurement made at a relative height of 30 percent, with the resultant *z* value applied to all trees, the between-stand error due to stem form would increase from .00155 to .00315 (the sum of the two variance components associated with method 4). The general formula for the resultant between-stand variance, including intrinsic (I) and sampling effects (S) is:

$$V(I+S)_{bs} = \sigma^2_{bs} + \sigma^2_{ws} / \sqrt{N}$$

where N is the number of trees with sampled upper stem *dob*'s. The above formula is exactly correct for a simplified situation. Consider that in summarizing a cruise, the mean of ln(VOLP3 / VOLP2) is to be calculated, where VOLP2 and VOLP3 are volume predictions based on the two-point and three-point sytems, respectively, and that the resultant mean value is to be used in adjusting the usual two-point estimate of volume for the stand; the final volume prediction for the stand will be the base volume estimate times the exponential of the mean ln(VOLP3 / VOLP2). The reader may recognize that these calculation are closely related to typical "vbar" calculations. But the point we wish to make is that the resultant calculation of stand volume (adjusted for the upper stem measurements) will be almost the same as though a mean *z* had been obtained for the upper stem measured trees and applied to all trees. This is easy to verify with cruise results. Assuming this assertion is correct, then the sampling variance for mean ln(VOLP3 / VOLP2) declines with the square root of N, and the variance formula must be correct. The reason we choose to summarize by mean

z rather than by mean ln(VOLP3 / VOLP2) is that in doing so we obtain more than just volume; we get fully compatible profiles for all trees and the ability to generate Scribner volumes or log assortment tables.

Measurement error in *dob* will cause a further increase in between-stand variance in the error for *ln*(VOL). That variance may be estimated by using the relationship between *dob* and predicted volume. The variance in ln(VOL) associated with measurement error in *dob* is:

$$\sigma^2_{me} = m^2 \times \sigma^2_{dob}$$

where m is coefficient reported in the previous section, and σ_{dob}^2 is the measurement variance for ln(*dob*) at the recorded height. For example, at a 10 percent relative height, with a 1 percent standard deviation in dob due to measurement error:

$$\sigma^2_{me} = 1.262^2 \times 0.0001 = 0.00016$$

Recall that the within-stand variance in ln(VOL) error (for a prediction using *dob* at 10 percent) is 0.00357; hence, the variance being added by a random 1 percent measurement error is relatively unimportant. The general formula for between-stand variation in volume error due to intrinsic differences in form (I), sampling for upper stem *dob*'s (S) and measurement error (ME) on the *dob*'s, is:

$$V(I+S+ME)_{bs} = \sigma^2_{bs} + \frac{\sigma^2_{ws} + m^2 \times \sigma^2_{dob}}{\sqrt{N}}$$

Predictions from the above equation are in table 3. In viewing that table, recall that a between-stand variance in error for ln(volume) of 0.00155 can be obtained without using any upper stem measurements. Thus, sampling for upper stem measurements using small sample sizes or large measurement errors will increase the variability of the error in estimating volume for a stand.

The majority of the INGY sample trees are of species that are not the most prevalent in their respective stands. Within each stand, the sample trees will tend to have a highly variable selection of tree sizes. These two facts would lead us to the suggestion that the σ_{ws}^2 values in table 2 are greater than would typically be seen in the primary species for single-age stands. We have a database that allows us to estimate σ_{ws}^2 within a 40-year-old Douglas-fir stand in the Willamette National Forest, about 50 miles west of Eugene, OR. In that stand, 44 representative trees were felled; inside bark and outside bark diameter measurements were made along the stem. We determined total volume for each tree, calculated predicted volumes based on DBH, HTOT, and dob at 33 feet. The percentage errors in volume were summarized: mean 0.5, standard deviation 3.4. This has an approximate correspondence to a σ_{ws}^2 value of 0.00116. This is about half of the estimated within-stand variance for method 4, as reported in table 2. Hence, it is possible that many important cruising situations will have within-stand variances much lower than indicated by the table.

Table 3.—*Between-stand variance in error for ln(volume), considering form differences, sampling error for upper stem dob and measurement error of upper stem dob. The measurement error refers to the standard deviation of the error in measuring ln(dob). N is the number of sample trees for which an upper stem dob is obtained.*

Rel. Ht (%)	Meas Err	N=8	N=16	N=32	N=64	N=128	N=infinity
10	0	.00246	.00209	.00183	.00165	.00152	.00120
	.01	.00252	.00213	.00186	.00167	.00153	.00120
	.03	.00297	.00245	.00208	.00183	.00164	.00120
	.05	.00387	.00309	.00253	.00214	.00187	.00120
20	0	.00183	.00153	.00132	.00117	.00106	.00081
	.01	.00189	.00157	.00135	.00119	.00108	.00081
	.03	.00238	.00192	.00160	.00137	.00120	.00081
	.05	.00336	.00262	.00209	.00171	.00145	.00081
30	0	.00164	.00140	.00122	.00110	.00102	.00081
	.01	.00169	.00143	.00125	.00112	.00103	.00081
	.03	.00211	.00173	.00146	.00127	.00114	.00081
	.05	.00296	.00233	.00189	.00157	.00135	.00081

DISCUSSION

The analysis of the variance components presented above can be coupled with expectations of precision and accuracy for various instruments to devise cruising plans that will lead to improved estimates of stand volume. In applying any taper system, validation of the system is to be recommended. However, the three-point system can be applied to populations different from those used in fitting without nearly as much risk as for a two-point system. With a three-point system, the biggest risk is that the upper stem dob's will be measured in a biased fashion.

References to measurement error for upper stem dob's are Schmid-Hass and Winzeler (1981), Winzeler (1986), Skovsgaard *et al.* (1998), and other papers in the present symposium. Precision depends on diameter and measurement height. For diameters of 30 cm at a height of 7 m. Schmid-Haas and Winzeler report standard deviations ranging from 1.3 percent for the Finnish caliper to 3.8 percent for the Relaskop. The cited publications all mention bias as a very serious problem; it can be due to individual operator error, instrument miscalibration, or systematic errors in the use of the instruments. In validation of field techniques, the use a paint-ball gun to mark a target height is recommended. This allows the two potential sources of error—due to height measurement and *dob* measurement—to be separately tracked. The measurement variance referred to in the previous section, σ_{dob}^2, is, of course, based on a comparison of the recorded diameter and the actual diameter at the recorded height (which is not necessarily at the paint mark).

Field techniques must be used correctly for any cruising or inventory situation. A good stem form model can help detect measurement biases. Using a profile model similar to the INGY model, we helped identify two sources of bias in USDA Forest Service field measurement procedures. Measuring upper stem diameters on standing trees is difficult, particularly at greater heights. In Forest Service tests, the Criterion laser was used to determine upper stem diameters; however, we used the instrument inappropriately. We had used the laser to calculate the horizontal distance to the face of the tree at DBH rather than to the vertical projection of the face of the tree at the upper bole measurement point. Because the calculations built into the laser assumed a horizontal distance to the face of the tree at the upper bole measurement point, the resulting answer had a consistent bias. The calculations done by the instrument resulted in an understatement of the upper stem diameter; adjusting the profile model for this understated upper bole diameter results in an apparent faster taper, resulting in an underprediction of volume.

Another problem exists with mixing and matching diameter tape and caliper measurements; for example, using a d-tape to measure DBH and an optical instrument (i.e., Caliper) to measure upper bole diameters. To determine the taper rate, both diameters should be taken with calipers or both should be taken with the d-tape; since a taped measurement of the upper bole is not practical, an additional caliper measurement should be used for taper rate calculation. We have data from a previous study where we took standing tree diameter measurements with a tape, mechanical calipers, Barr & Stroud dendrometer, and Criterion Laser; the mechanical caliper measurements were taken up the bole by climbing the tree. An analysis of the 300+ DBH measurements indicated that on the average, the taped diameter at breast height was slightly larger than the average caliper measurement. Again, using the taped DBH with a caliper upper bole measurement resulted in an overstatement of taper rate.

At the time of model development, and again prior to model implementation, some form of validation should be done to test the ability of the model to predict accurately and precisely. Where possible, validate a model by using data collected independently of data used to build the model. An important additional part of the validation process, particularly for two-point applications, should include the monitoring and re-evaluation of these models over time. This is necessary to have a continuing level of confidence that the models are producing reliable estimates of volume across the range of size and geographical areas they are being used in. Additionally, sampling techniques can be used to estimate the bias attributable to a taper system for a particular region.

The flexibility of the multi-point taper systems described here allows them to be used in several ways. These include:

1. As a two-point model—since most models are developed from data collected over broad areas, the models should predict profiles (and thus volumes) "on the average" very well throughout the area.

2. As a three-point (or multi-point) model—more accurate but involves additional measurements, thus more costly. One or more upper stem dob's are measured at predetermined heights on each tree being cruised; these measurements provide additional information for the model to predict profile, allowing individual tree profile differences to be recognized.

3. As a two-point model with a three-point subsample—a compromise between the first two options. Since it requires additional effort and cost to collect upper stem measurements, you may want to minimize the number of trees where they are taken; sampling, for example in the pre-cruise, could be used to calculate an average z value, or alternatively a three-point to two-point volume ratio, with the selected summary

applied to all trees in the cruise. It could be hypothesized that taper is a function of a tree's competative position in the stand, and therefore, the z score could be regressed on DBH. However, based on 2 years' experience with several forest types, it seems that a simple average for the stand is adequate.

4. With an assumed taper or z score—past experience may indicate that a particular stand type or subregion has a consistent taper, and an assumed z score could therefore be used.

The profile predictions are based on a system of equations; part of that system is a prediction of bark thickness at breast height. That prediction is, of course, reflective of the bark thicknesses observed in the original data set. In practice, we have found that there is substantial variability in double bark thickness at breast height, both between trees within stands, and between stands. Since bark thickness at breast height is fairly easy to measure and has the potential for large improvements in volume accuracy, it should be considered as a measurement variable. However, it is critical to verify that such measurements can be made without bias.

SUMMARY

The application of multi-point taper systems to cruising has been reviewed. The variance analyses suggest that fairly large numbers of trees must be measured for upper stem *dob* in order to reduce the between-stand variance in estimated volume. The requirement for large samples is driven by the high within-stand variance in stem form. Because of the way the data were obtained, that variance may be higher than will typically be found in a relatively homogeneous stand, with one or two major species. The supposition that smaller samples of upper stem dob's are adequate is supported by the low within-stand variance found in one Douglas-fir stand, and by the observation that significant mean z values can be found often with cruised samples of 30 to 40 trees. In measuring the upper stem *dob*'s, bias is to be carefully guarded against by validating measurement techniques and calibrating instruments. In addition to using upper stem dob's, there is a potential for using measured bark thickness at breast height.

The techniques described here work well with what we believe to be good taper systems. However, if the underlying two-point taper system has biases, the multi-point system has the ability to greatly reduce those biases. In some of the species we have worked with, there are hints of some small biases—namely a fairly consistent trend of negative mean z values for stands in some areas. If three-point taper cruising is used, the effect of any biases is greatly reduced. However, if the three-point cruising introduces measurement bias, it can change an unbiased prediction system into a biased prediction system.

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APPENDIX A.— Determining Diameter of a Standing Tree Anywhere Along the Bole Using a Relaskop

The Spiegal-Relaskop, and in fact any angle gauge, can be used to measure tree diameters at any height along the bole that is clearly visible. Measurement techniques that avoid interpolation on the scale may be the most precise. For example, usage of an American Scale Relaskop, using a 75.6 Basal Area Factor (BAF), located approximately in the middle of the fifth white bar to the right of "0" on the scale, is described. Attach a measuring tape to the center of the tree, and while viewing through the Relaskop, back away until the left side of the tree touches the "0" scale, and the right side of the tree touches the middle of the fifth white bar, or the equivalent of a borderline tree. At this point, the distance away from the tree in feet equals the diameter of the tree in inches. The Relaskop adjusts for slope, so the diameter can be measured at any height along the bole.

The following formulas show the calculation of the diameter using any BAF:

$$dia = \frac{dist}{PRF}$$

where :

dia is the diameter of the tree in inches

$dist$ is the distance to the tree in feet

$PRF = \frac{8.696}{\sqrt{BAF}}$ is the plot radius factor

When using the Relaskop, a convenient relationship of 1 inch diameter for each foot of distance from the tree results with a 75.6 BAF (midpoint of the fifth white bar); the calculation results in a PRF=1. The formulas allow you to use any BAF, although it is not a direct one-to-one relationship as with the 75.6 BAF. Also note that when using a prism, to adjust for slope, tilt the plane of the prism so that it is perpendicular to the line of site to the point of measurement on the bole.